2.5 Physically-based Animation
Physically-based animation

- Morphing allowed us to animate between two known states.
- Typically, only one state of an object is known.
- The known state is considered the equilibrium.
- The object is animated by applying physical laws.
Goals

- Animating and simulating
  - solid objects:
    - rigid objects
    - elastic objects
    - plasticity
    - fracture
  - fluids
  - gases
  - sheets: cloth draping
  - strings: hair
Deformable models
References

• Physically Based Deformable Models in Computer Graphics.
  Andrew Nealen, Matthias Müller, Richard Keiser, Eddy Boxerman, and Mark Carlson.
Rigid models

- Just affine transformations
- No problem, if dynamics are known
Dynamics

ACM © 1988 “Spacetime Constraints”
**Piecewise rigid models**

- Affine transformations with constraints
- No problem, if dynamics are known
Procedural animation

- Describes the motion algorithmically
- Expresses animation as a function of small number of parameters
- Example 1: a clock with second, minute and hour hands
  - Hands should rotate together
  - Express the clock motions in terms of a “seconds” variable
  - The clock is animated by varying the seconds parameter
- Example 2: A bouncing ball
  - \( \text{Abs}(\sin(\omega t + \theta_0)) \cdot e^{-kt} \)
Physically-based animation

- Assign physical properties to objects (masses, forces, inertial properties)
- Simulate physics by solving equations
- Realistic but difficult to control

\[ \text{mass} \quad \text{velocity} \quad \text{gravity} \]

\[ m \quad v_0 \quad g \]
Rigid body dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction
Collisions

- We know how to simulate bouncing really well
- But resting collisions are hard to manage
Collision detection

- Can become expensive
- Use fast collision checks with bounding volumes
Fracture

• Objects break apart when applied forces are too strong
• Fracture threshold
• Remeshing (need connectivity info)
• Material properties
• Parameter tuning
Deformable models

- Not just affine transformations.
- Object changes its shape.
2.6 Mass-spring Models
Mass-spring model

- The vertices of a mesh become particles.
- Particles are assigned a certain mass (typically uniformly).
- Forces are applied between the particles.
- The forces try to maintain the equilibrium, i.e., the undeformed state.
- Forces can be implemented by assigning a spring model to each edge of the mesh.
Hooke's law

\[ F = -kx \]
Spring forces

- Force in the direction of the spring and proportional to difference with rest length

\[ F(P_i, P_j) = K(L_0 - \|P_i P_j\|) \frac{P_i P_j}{\|P_i P_j\|} \]

- K is the stiffness of the spring
  - When K gets bigger, the spring really wants to keep its rest length
How would you simulate a string?

- Each particle is linked to two particles
- Forces try to keep the distance between particles constant
- What force?
How would you simulate a string?

- Springs link the particles
- Springs try to keep their rest lengths and preserve the length of the string
- Not exactly preserved though, and we get numerical oscillation
  - Rubber band approximation
Mass-spring models

- Interaction between particles
- Create a network of spring forces that link pairs of particles
- Used for strings (hair, ...) and sheets (cloth, ...)

![Diagram of mass-spring model](attachment:image.png)
Mass-spring models

- Can generate 1D, 2D, and 3D connectivity
Three types of forces

- **Structural forces**
  - Try to enforce invariant properties of the system
  - E.g. force the distance between two particles to be constant
  - Ideally, these should be constraints, not forces

- **Internal deformation forces**
  - E.g. a string deforms, a spring board tries to remain flat

- **External forces**
  - Gravity, etc.
Spring-mass systems

- The state of a system at a given time $t$ is defined by the positions $x$ and velocities $v$ of all the masses $m$.
- The force $f$ of each mass is computed due to
  - its spring connection with its neighbors
  - its external forces
- The motion is governed by Newton's second law
  \[ f_i = m_i \ddot{x}_i \]
- For the entire system, we get
  \[ M\ddot{x} = f(x, v) \]
- $M$ is a diagonal matrix.
- Solve the ordinary differential equation using numerical integration.
Linear elasticity

- Commonly, springs are modeled as being linearly elastic:

\[ f_i = k_s (|x_{ij}| - l_{ij}) \frac{x_{ij}}{|x_{ij}|} \]
**Viscoelasticity**

- Physical bodies are not perfectly elastic.
- Viscoelastic springs are used to damp out relative motion.
- In addition to the elastic force, each spring adds a viscous force by looking into velocity differences:
  \[ f_i = k_d (v_j - v_i) \]
- Unfortunately, this damps rigid body rotations as well as bending and wrinkling.
- Better: project velocity difference onto the edge to restrict force along that direction:
  \[ f_i = k_d \left( \frac{v_{ij}^T x_{ij}}{x_{ij}^T x_{ij}} \right) x_{ij} \]
  \[ v_{ij} = v_j - v_i \]
Plasticity

• Plasticity can also be modeled by setting the force to
  - a constant negative value, if the distance is smaller than the equilibrium
  - a constant positive value, if the distance is larger than the equilibrium
  - zero, if the distance is the equilibrium
• This is numerically unstable, but can be coupled with elastic and viscous components.
Spring forces

- elastic, plastic, and viscous forces:

\[ f_e \]

\[ r_0 \]

\[ ||d_y||_2 \]

(a)

\[ f_e \]

\[ r_0 \]

\[ ||d_y||_2 \]

(b)

\[ f_e \]

\[ \frac{d_y^T \dot{d}_{ij}}{||d_y||_2} \]

(c)
Deformation energies

- More general solutions define deformation energies.
- Various energies can be embedded.
- Constraints of the form $C(x) = 0$ can be embedded using the energy term

$$\frac{k_s}{2} C^T(x) C(x)$$

- These energies are minimized.
- They can enforce preservation of distances, angles, areas, volumes, etc.
- Forces are computed with respect to positions

$$f_i = -\frac{\partial E}{\partial x_i}$$
Lennard-Jones function

- The Lennard-Jones function is widely been used for spring model energies.
- It represents a potential energy

\[ f_{ij} := -\frac{\partial E_{\text{pot}}(\|q_j - p_i\|_2)}{\partial p_i} \]

- It is given by:

\[ E_{\text{pot}}(r) = E_{\text{LJ}}(r) := \frac{B}{r^s} - \frac{A}{r^t} \]

\( B > A > 0 \)
\( s > t > 0 \)

\[ -\frac{dE_{\text{LJ}}(r)}{dr} = \frac{SB}{r^{s+1}} - \frac{tA}{r^{t+1}} \]

\[ -\frac{dE_{\text{LJ}}(r)}{dr} = 0 \quad \iff \quad r = r_0 := \sqrt[1-s]{\frac{SB}{tA}} \]
2.7 Example for Strings: Hair Animation
Example: Hair

- 1D mass-spring model
Reference

- **Realistic hair simulation: animation and rendering**
  Florence Bertails, Sunil Hadap, Marie-Paule Cani, Ming Lin, Tae-Yong Kim, Steve Marschner, Kelly Ward, and Zoran Kacic-Alesic
• Add additional springs:
Hair

- Each linear spring affects both bending and stretching.
- Ambiguity of linear springs:
Hair

- Use angular springs instead.
- Deformation forces proportional to the angle between segments
Hair

• Energies (straight hair):
  - linear springs
    \[ E = \frac{1}{2} k \sum (l - l_{\text{rest}})^2 \]
  - angular springs
    \[ E = \frac{1}{2} k \sum \theta^2 \]

• Discretization:
  - more segments imply higher stiffness.
Hair

• Animation of hair movement under changing external force can be run with thousands of hairs in less than a second per frame.
• However, hairs would penetrate each other, which does not lead to a realistic overall animation.
• Hair-hair interaction must be considered.
Hair collision

- Bounding cylinders

- Swept sphere volume
Hair collision

- Animation w/o and w/ hair-hair interaction
Hair strands

- Resolving hair-hair interaction is very expensive.
- Idea: group hairs together to strands.
Multiresolution hair representation

- Multiresolution strand representation
Multiresolution hair representation

- Multiresolution representation

- Switch to higher resolution based on
  - Visibility
  - Viewing distance
  - Hair motion: high velocities
Multiresolution hair representation

• Multiresolution hair simulation
Multiresolution hair representation

- Merging of strands

(a) children skeletons
(b) parent skeleton
(c) first distance threshold
(d) second distance threshold
Video

- Multiresolution hair animation.
Hair in Shrek 3

- single strand
- very stiff initial section
Hair in Shrek 3

- adjust rest shape to account for shape change due to gravity
Hair in Shrek 3

- extensive collision resolving
Hair in Chronicles of Narnia

- stiff hair
- clumping together