5.2 Blinn-Phong Reflection Model
Blinn-Phong reflection model

- The Blinn-Phong reflection model is a slight modification of the Phong reflection model.
- The purpose of the modification is to speed up the computation.
- The modification only affects the specular reflection.
- The faster computation avoids the expensive computation of the reflected direction $r$ of the light ray.
Blinn-Phong reflection model

- It introduces the half-way vector
  \[ h = \frac{e + v}{\|e + v\|} \]

- The specular highlight is brightest, if \( v = r \), i.e., if \( n = h \).
- The angle \( \beta \) between \( n \) and \( h \) can be used to compute the falloff of specular intensity.
Blinn-Phong reflection model

- The Blinn-Phong reflection model replaces $\max (r \cdot v, 0)$ with $\max (h \cdot n, 0)$.

- We obtain

$$I = I_a \cdot k_a + \sum_i I_p \cdot f_{\text{att}} (s_i) \cdot \left[ k_d \cdot \max (n \cdot e_i, 0) + k_s \cdot \max (h \cdot n, 0)^n \right]$$
Blinn-Phong vs. Phong

• The two models are not equal.
• By adjusting the specular reflection exponent $n$, one can obtain results with both models that look alike.
5.3 BRDF
Motivation

• Phong’s diffuse reflection is based on the Lambertian model.
• The reflection is equal in all directions.

• It represents perfectly diffuse (matte) surfaces.
• How can we model other surface reflections?
Bidirectional reflectance distribution function:

- BRDF is a 4-dimensional function that defines how light is reflected at an opaque surface.
- The function takes an incoming light direction and outgoing direction, both defined with respect to the surface normal, and returns the ratio of reflected radiance exiting in the outgoing direction to the irradiance incident on the surface from incoming direction.
BRDF

- Each direction is parameterized by azimuth angle and elevation, therefore the BRDF as a whole is 4-dimensional.
BRDF

Definition:

- Formally, a BRDF can be described by

\[
 f_r(\omega_i, \omega_o) = \frac{dL_r(\omega_o)}{dE_i(\omega_i)} = \frac{dL_r(\omega_o)}{L_i(\omega_i) \cos(\theta_i) d\omega_i}
\]

where \(L\) is the radiance, \(E\) is the irradiance, and \(\theta_i\) is the angle between \(\omega_i\) and the surface normal.
**BRDF**

Rules:

- Helmholtz reciprocity

\[ f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i) \]

- Energy conservation

\[ \forall \omega_i, \int_{\Omega} f_r(\omega_i, \omega_o) \cos \theta_o d\omega_o \leq 1 \]
Theoretical models

• Lambertian model:
  - constant BRDF

• Phong & Blinn-Phong model:
  - Lambertian + specular reflectance
Theoretical models

• He’s model:
  - BRDF is the result of fine-scale roughness of the surface.
  - Collection of microfacets with random sizes and orientations.
  - Each facet is taken to be a perfect reflector.
Simulated models

- **Westin:**
  - Simulation to model complex microgeometry.
  - Non-isotropic, i.e., reflection not independent of azimuth.
  - Models many materials including velvet and brushed aluminium.
Isotropic vs. anisotropic reflection
Experimental models

- Measure reflectance properties:
5D BRDF

• For several materials, the BRDF depends on the wavelength of the incoming light.
• Wavelength can be added as another dimension.
• Example: aluminium

\( \lambda = 2 \mu \text{m} \)

\( d = 500 \text{ nm} \)
7D BRDF

• BRDF may also depend on the position on the surface.
• Using a 2D parametrization of the surface, this adds another two dimensions, leading to a 7D BRDF model.

\[
BRDF_\lambda(\theta_i, \phi_i, \theta_o, \phi_o, u, v)
\]

• \(\lambda\) is wavelength
• \(\theta_i\) and \(\phi_i\), represent the incoming light direction in spherical coordinates
• \(\theta_o\) and \(\phi_o\) represent the outgoing reflected direction in spherical coordinates
• \(u\) and \(v\) represent the surface position parameterized in texture space
6. Shading
6.1 Flat Shading
Shading of polygonal meshes

- Given a polygonal mesh, we can evaluate the illumination formula for each polygon.
- Example:

For each triangle \( pqr \), the normal \( \mathbf{n} = (\mathbf{q} - \mathbf{p}) \times (\mathbf{r} - \mathbf{p}) \) is constant.
Flat shading

- Assuming that light rays are hitting a triangle in an almost parallel fashion, the illumination within a triangle is constant.
- Hence, a uniform color is assigned to each triangle.
- This is referred to as flat shading.
Flat shading

- Flat shading produces the effect of a flat polygon.
- It is suitable to render planar surfaces.
- Often, polygonal surfaces are a piecewise linear approximation of curved surfaces.
- In this case, flat shading is not suitable for photorealistic rendering and should be replaced by smooth shading.
Flat shading
Mach band effect

- The Mach band effect increases the visual unpleasant representation of curved surface using flat shading.
Mach band effect

- The Mach band effect describes the phenomenon that the contrast between adjacent unicolored areas appear higher than actually present due to the human visual apparatus that focuses on extracting contrast.
Enhanced contrast
Also true for colored images
What happens when contrast is missing?
Back to triangles
Smooth shading
6.2 Gouraud Shading
Gouraud shading

- Gouraud shading computes a smooth continuous shading for surfaces represented by triangular meshes.
- Each pixel of the projected triangle gets assigned a color individually.
Outline

1. Apply illumination model at the vertices of the triangular mesh and compute the respective color.
2. Using the colors at the vertices of a triangle, interpolate the color values at the interior points of the triangle.

Remark: The interpolation can be executed in screen space, i.e., after the triangle has been projected.
Smooth shading

(a1)  

(b1)  

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Normal estimation

• If the triangular mesh is an approximation of a known analytical function, the function can be evaluated at the respective position.
Normal estimation for triangular meshes

- Given all normals of the adjacent triangles, the normal at a vertex of a triangular mesh can be computed as the average of the triangles' normals.

\[ \mathbf{n} = \frac{1}{k} \sum_{i=1}^{k} \mathbf{n}_i \]

where \( k = \# \text{ neighbors} \)
Normal estimation for triangular meshes

- As the triangles are not of equal size and shape, the average can be weighted accordingly.

- Example:
  - Weights defined by adjacent angles:

\[
\hat{\mathbf{n}} = \frac{\sum_{i=1}^{k} \alpha_i \mathbf{n}_i}{\sum_{i=1}^{k} \alpha_i}
\]
Normal estimation by local fitting

- Normals can also be estimated by locally fitting a surface through the points.
- Fitting is typically approximate using some kind of minimization such as the least-squares method.
- One may fit polynomial surfaces of higher degree.
- It typically suffices to fit a plane and take its normal as an estimate (see Section 2.3).
Interpolation

Barycentric coordinates:

- Once the color values are known at the vertices of a triangle, the color values in the interior can be computed using barycentric coordinates.

\[ \mathbf{p} = \alpha_1 \mathbf{p}_1 + \alpha_2 \mathbf{p}_2 + \alpha_3 \mathbf{p}_3 \]

\[ J(\mathbf{p}) = \alpha_1 J(\mathbf{p}_1) + \alpha_2 J(\mathbf{p}_2) + \alpha_3 J(\mathbf{p}_3) \]
Interpolation using scanline algorithm

• Idea:
  - Exploit knowledge about already computed color values.
  - Traverse projected triangle top-down using scanline.
  - Compute start and end color value of each pixel row using linear interpolation along the triangle’s edges.
  - Fill each pixel row using linear interpolation between the start and end color of that row.
Interpolation using scanline algorithm

\[ J(q_i) = J(p_i) \frac{y_3 - y_{p_i}}{y_2 - y_1} + J(p_i) \frac{y_0 - y_{p_i}}{y_3 - y_1} \]

\[ J(q_1) = J(p_1) \frac{y_3 - y_{p_1}}{y_2 - y_2} + J(p_3) \frac{y_{p_1} - y_1}{y_3 - y_2} \]

\[ J(p) = J(q_1) \frac{x_i - x_{p_1}}{x_2 - x_1} + J(q_1) \frac{x_{p_1} - x_4}{x_1 - x_4} \]
Gouraud shading