HOMEWORK ASSIGNMENT 3 (THEORY)

CO19-320322: COMPUTER GRAPHICS
320322: GRAPHICS AND VISUALIZATION

Fall 2016

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Due: Friday, October 7, 2016, at 8pm.

Problem 3: Object Representation & Ray Casting *(4+7+4=15 points)*

(a) *Surface reconstruction.* Consider the 2D analogon of a (boundary) surface reconstruction, i.e., a (boundary) curve reconstruction. Assume that the curve has been sampled with some measuring device and the acquired points on the curve are given by $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (2, 3)$, $\mathbf{v}_3 = (3, 1)$, and $\mathbf{v}_4 = (4, 3)$. Reconstruct the curve by performing the following steps:

- Draw the 2D Voronoi diagram of the four points as precisely as possible (no need to perform computations though).
- Compute the (2D) Delaunay triangulation from the Voronoi diagram.
- Use the α-shapes approach to generate the bounding curve from the Delaunay triangulation. Which α did you use? Show clearly what the output is and explain why.

(b) *Ray casting of triangular fan.* Given a triangle fan with vertices $\mathbf{v}_1 = (0, 4, 2)$, $\mathbf{v}_2 = (0, 0, 2)$, $\mathbf{v}_3 = (4, 0, 2)$, and $\mathbf{v}_4 = (5, 5, 2)$ in 3D Cartesian coordinates. In the same coordinate system, we define a view point $\mathbf{p} = (0, 0, -10)$ and the screen by the quadrilateral $\mathbf{v}_1 = (-2, -2, 0)$, $\mathbf{v}_2 = (2, -2, 0)$, $\mathbf{v}_3 = (2, 2, 0)$, and $\mathbf{v}_4 = (-2, 2, 0)$. The resolution of the screen shall be 2×2 pixels. The first triangle of the triangle shall have yellow color, the second green, while the background shall be black. Apply a ray-casting approach to the triangle fan by executing the following steps:

- Compute a parametric representation of each of the rays.
- Compute an implicit representation of the planes that contain the triangles of the triangle fan.
- Compute the intersections of the rays with the planes.
- Check whether the intersection points lie inside the triangle using barycentric coordinates. (Note: If rays and triangles obviously cannot intersect, you do not need to test them, but explain in a mathematically sound way why they cannot intersect.)
- Check whether the intersection points lie inside the triangle using the alternative algebraic approach. (Note: If rays and triangles obviously cannot intersect, you do not need to test them, but explain in a mathematically sound way why they cannot intersect.)
- For each pixel, report back the correct color obtained by the ray-casting approach.

(c) *Ray casting of ellipsoid.* Given an ellipsoid centered at point $\mathbf{o} = (0, 1, 0)$ whose axes are aligned with the three coordinate axes and the radii being 2, 2, and 1, respectively. Given viewpoint $\mathbf{v} = (0, 0, -2)$, Compute the intersection of a ray from the viewpoint $\mathbf{v} = (0, 0, -2)$ through a pixel with center $\mathbf{c} = (0, 0, 0)$ with the ellipsoid by performing the following steps:

- Compute a parametric representation of the ray.
- Compute an implicit representation of the ellipsoid.

- Compute the intersections of the ray with the ellipsoid.
- Return the first intersection point of the ray with the ellipsoid.

Remarks: The theoretical assignments have to be submitted in paper form into the box labeled “Linsen” in the Research I entrance hall. In case the theoretical part is typed (e.g., using LaTeX), the generated PDF-file can also be uploaded to jGrader.