Computational Electromagnetics

Introduction

Dr. Wallace

Fall 2009
Outline

Background

Lecture Description

Laboratory Description
Why study electromagnetics (EM)?

- Reason #1: It's cool!
  - Waves and fields ≈ magic!
  - Action at a distance.
  - What are fields? How do they work?
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▶ Electric, electrical, optical devices governed by EM
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- Applications of EM: Satellite comm., wireless, GPS, radar, optical storage, fiber-optic networks, ...
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What happens when simplifying assumptions break down (V=0/1, prop. delay model, no cross-talk, etc.)?
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▶ What happens when simplifying assumptions break down (V=0/1, prop. delay model, no cross-talk, etc.)?
▶ Knowing EM: Can develop new models.
Why study electromagnetics (EM)?

Reason #3: It fosters mathematical maturity.

Electromagnetics applies:
- Multivariate calculus
- Vector algebra and calculus
- Partial differential equations
- Fourier transform theory
- Complex analysis
- Asymptotic analysis
- Linear algebra
- Statistics

A good way to become comfortable with these!
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What is computational EM?

Maxwell’s Equations

\[ \nabla \cdot \mathbf{D} = \rho \]
\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \]
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- No best CEM method: Generality, accuracy, efficiency, ease of implementation
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Concepts covered: 1. Basic numerical techniques

Needed in other methods
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- Numerical integration
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- Random number generation
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- Numerical integration
- Random number generation
- Monte-Carlo methods
Concepts covered: 2. Finite-difference (FD) methods

\[ \frac{\partial f(x)}{\partial x} \approx \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} \]

- Discretize PDEs directly
Concepts covered: 2. Finite-difference (FD) methods

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- Advantages
  - Conceptually simple
  - Easy to implement

Disadvantages
- Derivatives amplify errors (accuracy)
- Volumetric method (high model order)

But, still used in practice!
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- Finite-difference time-domain (FDTD)

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Concepts covered: 3. Method of Moments (MoM)

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- Very efficient and accurate!
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- Cast PDEs into integral form analytically
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- Only unknowns on *surface* need be modeled
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- Very efficient and accurate!
- Price: extra analytical work
Concepts covered: 4. Variational Methods

\[ \nabla^2 \Phi(x, y) = -f(x, y) \]

\[ I = \frac{1}{2} \int \int_S [\Phi_x^2 + \Phi_y^2 - 2f(x, y)\Phi]dS \]
Concepts covered: 4. Variational Methods

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- Generalization of the moment method
- Convert the PDE into minimization of an integral equation
- Concepts:
  - Variational calculus
  - Converting PDEs to variational form (and back)
  - Rayleigh-Ritz method
  - Weighted residual method
Concepts covered: 5. Finite-element Method (FEM)

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- Solve resulting minimization problem
Progress still being made in CEM!

- Ray-tracing and hybrid methods
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- Finite-volume generalizations of FDTD (CST)
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<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Intro</td>
</tr>
<tr>
<td></td>
<td><strong>Basic Numerical Techniques:</strong> Integration, Monte Carlo Methods</td>
</tr>
</tbody>
</table>
| 2    | **FD Methods:** Laplace Equation  
      | Example: Transmission Lines |
| 3    | 1D Wave Equation, Absorbing Boundaries  
      | Stability of FD Solutions |
| 4    | Waveguides; Modal Solutions |
| 5    | **FDTD:** Yee cells, discretization, algorithm  
      | Materials, Sources, scattering  
      | Post processing, 2D FDTD |
| 6    | Perfectly Matched Layer (PML) |
| 7    | *Reading Day*  
      | *Midterm* |
| 8    | **Method of Moments:** Green’s Function, Laplace Eq.  
      | Capacitance Computation |
| 9    | Scattering problems, Far-Fields  
      | Radiation problems |
| 10   | Volumetric MoM |
| 11   | **Variational Methods:** Variational Calculus  
      | Rayleigh Ritz Method  
      | Eigenvalue problems, Weighted residuals |
| 12   | **Finite-Element Method (FEM)**:  
      | Element Assembly |
| 13   | **Modern Developments:** Ray-tracing; Hybrid Methods  
      | Fast Multipole Method |
| 14   | **Finite-Volume Time Domain**  
      | *Project Presentations* |
Prerequisite Material

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- Engineering math: Multivariate calculus, differential equations, Fourier transforms
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Computational Electromagnetics
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Grading

Weight of grade
  Homework assignments  20%
  Midterm  40%
  Final  40%
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Purpose of Laboratory
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▶ Practical experience implementing algorithms
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- How do you learn CEM without the C?
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- Designed to take less than 3 hours
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- Use own computer
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▶ About one assignment per week
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▶ Use own computer
▶ Any programming environment fine (Matlab, Octave, etc.)
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- Project
Lab Grading

Weight of grade
Lab Assignments  70%
Project Writeup   15%
Project Present  15%
Thank you

See you next time!