Chapter 3: Antenna Parameters
Introduction

Purpose
Introduce standard terms and definitions for antennas
Need a common language to specify performance

Two types of parameters
1. Radiation parameters
   *What is the spatial selectivity of the element?*
   *Indicate where is power sent / collected from.*

2. Network parameters
   *What does the antenna present at its port(s)?*
   *Indicates requirements for system it connects to.*
Outline: Radiation Parameters

Goal
Precisely define the spatial selectivity of antennas

Main Concepts
Radiation patterns, pattern cuts, beamwidth
Field regions: far-field, near-field
Power density of EM fields
Radiation Power Density
Directivity / Gain
Radiation Patterns

Definition
Graphical representation of radiation (or reception) properties
Function of spatial coordinates

Possible quantities
Power density (most common)
Field strength
Directivity
Gain
Phase
Polarization
Far-field / Cuts

Far-Field Patterns

Usually more interesting than near fields
Pattern only a function of angles ($\theta$, $\phi$)

Field Cuts

Complete 3D pattern difficult to visualize (and plot!)
More precise to look at cuts of the pattern:
Far-field / Cuts: Patch Antenna

3D Pattern

xz and yz cuts

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Earth Coordinate System

Global Coordinate System
- Horizontal (H) / Azimuth
- Vertical (V) / Elevation

Caution
- Depends on how antenna is mounted
- Natural coordinates for analyzing antenna (x, y, z)
- May be different from way mounted relative to Earth
- Need to rotate axes
General Pattern Types

Isotropic Pattern
- Power (or field) equally radiated in all directions
- In practice, does not exist!
- Used as a reference

Omnidirectional Pattern
- Radiated field constant in azimuth ($\phi$)
- May vary with elevation ($\theta$)
- Examples: dipole or small loop

Directional Pattern
- Radiates significantly more power in some directions than others
  “Directional in the ________ plane”
- Significantly more directional than a half-wave dipole
Principal Patterns

Motivation
Defines patterns independent of coordinate system
Useful for antennas with linear polarization

E-Plane Pattern
Cut of the pattern containing $\mathbf{E}$ and the direction of max radiation

H-Plane Pattern
Cut of the pattern containing $\mathbf{H}$ and the direction of max radiation
Principal Patterns

Example: Horn Antenna
Beamwidth

Definition

Angular extent of the main beam

Criteria

HPBW: Half-power beamwidth

FNBW: First null beamwidth
Field Regions

Reactive Near-field

Region immediately surrounding antenna
Convention: \[ R < 0.62 \sqrt{\frac{D^2}{\lambda}} \]
Fields can be very intense
Mostly reactive (stored energy, not propagating)

Note:
- \( D \) = largest antenna dimension
- \( \lambda \) = wavelength

Caution

Expressions do not work for electrically small antennas
Maximum dimension must be comparable or larger than \( \lambda \)
Field Regions (2)

Radiating Near-Field (Fresnel) Region

- Fields are radiating
- But, radiation pattern is a strong function of distance $r$
- Convention: $0.62 \sqrt{D^3/\lambda} \leq R < 2D^2/\lambda$

Far-Field (Fraunhofer) Region

- Angular field distribution nearly independent of distance
- Fields are transverse to direction of propagation
- Convention: $R \geq 2D^2/\lambda$
Power Flow

Power Flow of EM Field

Instantaneous Poynting Vector

\[ \overline{W}(t) = \overline{E}(t) \times \overline{H}(t) \]

\( \text{W/m}^2 \quad \text{V/m} \quad \text{A/m} \)

Time-average power

\[ < \overline{W}(t) >_t = < \overline{E}(t) \times \overline{H}(t) >_t \]

In frequency domain, this becomes

\[ \overline{W} = \frac{1}{2} \text{Re} \{ \overline{E} \times \overline{H}^* \} \]

Interpretation

Power per unit area ⇒ power density
Direction is direction of power flow
Power Radiated by Antenna

Total radiated power
Integrate over surface enclosing antenna

\[ P_{\text{rad}} = \iiint_S \mathbf{W}(\mathbf{r}) \cdot \hat{n} \, dS \]

\( \mathbf{w} \) on surface of \( S \)
- Power *radiated* per unit area
- Radiation power density

Visualization
- Generally fix \( r \) and plot \( \mathbf{W}(\theta, \phi) \)
Normalization

Can normalize $W$

$$U(\theta, \phi) = r^2 W(r, \theta, \phi)$$

Obtain power per unit solid angle
Independent of distance from antenna
$U$ is called radiation intensity

In Far-Field Region

$$\mathbf{H} = \frac{1}{\eta} \mathbf{\hat{r}} \times \mathbf{E}$$

$$\mathbf{W}(\mathbf{r}) = \frac{1}{2} \text{Re} \left\{ \mathbf{E} \times \mathbf{H}^* \right\} = \frac{1}{2} \text{Re} \left\{ \mathbf{E} \times \eta^{-1} (\mathbf{\hat{r}} \times \mathbf{E}^*) \right\}$$

$$dA = r^2 \sin \theta d\theta d\phi$$

Solid Angle

$$U(\theta, \phi) = \frac{r^2 |\mathbf{E}(r, \theta, \phi)|^2}{2\eta}$$
Directivity

Definition

Sometimes called “directive gain”, given by

\[ D(\theta, \phi) = \frac{U(\theta, \phi)}{U_0} = \frac{U(\theta, \phi)}{P_{\text{rad}}/(4\pi)} \]

Radiation intensity of given antenna
Radiation intensity of a reference antenna

Note: Total radiated power same for two antennas
Reference Antenna: Standard is to choose isotropic radiator

In terms of radiation density

\[ D(\theta, \phi) = \frac{|W(r, \theta, \phi)|r^2}{P_{\text{rad}}/(4\pi)} = \frac{|W(r, \theta, \phi)|}{P_{\text{rad}}/(4\pi r^2)} \]
Directivity (2)

Maximum Directivity

\[ D = \max_{(\theta, \phi)} D(\theta, \phi) = \frac{U_{\text{max}}}{P_{\text{rad}}/(4\pi)} \]

When directivity given as a single number \( \Rightarrow \) Maximum directivity

Notes

Directivity of an isotropic radiator is 1
Therefore, \( D > 1 \) in practice
\( D \) usually expressed in dB
Directivity (3)

Explicit Computation

Given far E-fields,

\[ D(\theta, \phi) = 4\pi \frac{|\bar{E}(\theta, \phi, r)|^2}{\int_0^{2\pi} \int_0^{\pi} |\bar{E}(\theta', \phi', r)|^2 \sin \theta' \ d\theta' d\phi'} \]

Observation:

*Directivity is the radiation density divided by the average radiation intensity (over solid angle)*
Gain

Comparison with Directivity

Directivity/Gain

\[
\text{Radiation intensity of given antenna} \quad \frac{\text{Radiation intensity of a reference antenna}}{}
\]

**Directivity:**
Total radiated power of two antennas kept the same

**Gain:**
\textit{Input} power of two antennas kept the same

What is the difference?

**Losses**
Gain (2)

Computation

\[
G(\theta, \phi) = \frac{U(\theta, \phi)}{P_{\text{in}}/(4\pi)} - e_t D(\theta, \phi)
\]

Efficiency

\(e_t\) is the total efficiency of the antenna

\[e_t = e_r e_c e_d\]

where

\(e_r = \) Reflection efficiency \(1 - |\Gamma|^2\)

\(e_c = \) Conduction efficiency

\(e_d = \) Dielectric efficiency

Radiation Efficiency

\[e_t = \frac{P_{\text{rad}}}{P_{\text{in}}} = \frac{R_{\text{rad}}}{(R_{\text{rad}} + R_L)}\]
Antenna Polarization

**Definition**

TX: Polarization of the radiated wave produced by the antenna

RX: Polarization of incident plane wave yielding maximum available output power at the antenna terminals

**Directional Dependence**

Polarization can be defined

1. As a function of direction
2. For direction of maximum gain
   (assumed if no direction specified)
**Review of EM Polarization**

**Definition**

For a plane wave propagating in the $-\hat{z}$ direction, the instantaneous field is

$$\overline{E}(z,t) = \hat{x}E_x(z,t) + \hat{y}E_y(z,t)$$

where

- \(E_x(z,t) = \text{Re} \left\{ E_{x0}e^{j(\omega t + kz + \phi_x)} \right\} \)
- \(E_y(z,t) = \text{Re} \left\{ E_{y0}e^{j(\omega t + kz + \phi_y)} \right\} \)

Maximum Amplitude of x,y Components

Phase of x,y Components

Polarization = Shape of curve traced by tip of E vector in xy plane

Direction of Propagation
Review of EM Polarization (2)

In xy plane

\[ \overrightarrow{E}(t, z = 0) = \hat{x}E_{x0}\cos(\omega t + \phi_x) + \hat{y}E_{y0}\cos(\omega t + \phi_y) \]

Traces out an ellipse in general

Special Cases

Linear polarization

\[ \Delta \phi = \phi_y - \phi_x = 0, \pi \]

\[ \overrightarrow{E}(t) = (\hat{x}E_{x0} \pm \hat{y}E_{y0})\cos(\omega t) \]

Circular Polarization

\[ E_{x0} = E_{y0} \]

\[ \Delta \phi = \phi_y - \phi_x = \begin{cases} +\frac{\pi}{2}, & \text{CW (right hand circular)} \\ -\frac{\pi}{2}, & \text{CCW (left hand circular)} \end{cases} \]
Outline: Network Parameters

Goal

Precisely define the “input/output interface” of the antenna

Main Concepts

Input impedance
Reflection coefficient / VSWR
Mutual Coupling, Z-parameters, S-parameters
Antenna Input Characteristics

Input Impedance

Have seen that mismatch reduces efficiency of antenna system

To ensure maximum transmission,
Conjugate match condition: \( Z_L = Z_g^* \)

In Practice

Antennas designed to have convenient input impedance (50 Ohms)

Matching network integrated in antenna
Transforms raw antenna impedance to \( Z_0 \)
Antenna Input Characteristics (2)

Nominal input impedance is $Z_0$
Actual impedance varies slightly with frequency
Also, no fabrication process is perfect
Variations in impedance from one antenna to next

Characterizing input Impedance
Graphical representations
Antenna Input Characteristics (3)

Problem with Providing Input Impedance

Impedance varies from one device to the next
(fabrication variations)
Every antenna must be measured

More common approach

Antenna design assumes system impedance of $Z_0$
Specify:
1. Worst case reflection, or
2. Voltage standing wave ratio (VSWR)
Reflection Coefficient

Definition

\[ \Gamma = \frac{V^-}{V^+} \text{ on the feeding line} \]

\[ |\Gamma|^2 \text{ indicates what fraction of power is reflected} \]

Power lost, because not delivered to antenna

Return Loss

Related to \( \Gamma \):

\[ \text{Return Loss} = -20 \log_{10} |\Gamma| \]

IEEE definition: Return Loss as a positive value (hence – sign)

Worst case return loss

\[ \text{Return Loss}_{\text{min}} = -20 \log_{10} |\Gamma|_{\text{max}} \]
VSWR

**Definition**

Voltage standing wave ratio
(max voltage to min voltage on feed line)

\[
V_{SWR} = \frac{|V(z)|_{\text{max}}}{|V(z)|_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}
\]

**Reason:**
Wave ratio was easy to measure with old slotted waveguides
Still used in many specifications of RF parts / antennas

Expressed as a ratio
i.e. 1.2:1 or 2:1
If a single number, indicates worst-case value
Mutual Coupling

Where important
- Antenna arrays
- Multimode or multipolarization antennas ⇒ Multiple ports

Basic problem
- Antenna elements close together
- Signals on one element ⇒ create signal on other element
- Usually want to receive signals on antennas independently

SP Algorithms
- Typically are degraded by the effect
Characterizations: Z-Parameters

Example: Dual Polarization Patch Antenna

- Square patch at 2.44 GHz
- Feed 1 ⇒ Vertical Pol.
- Feed 2 ⇒ Horizontal Pol.

Network Characterizations

1. Z-Parameters

\[
\begin{bmatrix}
  v_1 \\
  v_2 \\
\end{bmatrix}
= \begin{bmatrix}
  Z_{11} & Z_{12} \\
  Z_{21} & Z_{22} \\
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
\end{bmatrix}
\]

\[
\bar{u} = \bar{Z} \bar{\mathbf{i}}
\]

Coupling means \( Z_{21} \) or \( Z_{12} \) ≠ 0
Simulated Z-Parameters

- **Z11**
  - real part
  - imaginary part
  - freq. GHz

- **Z12**
  - real part
  - imaginary part
  - freq. GHz

- **Z21**
  - real part
  - imaginary part
  - freq. GHz

- **Z22**
  - real part
  - imaginary part
  - freq. GHz
Characterizations: S-Parameters

Network Characterizations

2. S-Parameters (S = “scattering”) More useful for high-freq. analysis

\[
\begin{bmatrix}
  v_1^- \\
  v_2^-
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  v_1^+ \\
  v_2^+
\end{bmatrix}
\]

\[
S_{ij} = \left. \frac{v_j^-}{v_i^+} \right|_{v_k^+=0, k\neq i}
\]

You should see relation to \( \Gamma \)

Worst-case coupling: \( 20 \log_{10} |S_{21}|_{\text{max}} \)

Often quoted as minimum isolation: \( -20 \log_{10} |S_{21}|_{\text{max}} \)
Simulated S-Parameters

**S11**

**S12**

**S21**

**S22**

freq, GHz

dB(S(1,1))

dB(S(1,2))

dB(S(2,1))

dB(S(2,2))

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Antenna Bandwidth

Definition
Range of frequencies over which the antenna conforms to some specified standard
“Specified standard” includes any performance metrics so far:

- Patterns
- Gain
- Efficiency
- Side lobe levels
- Beamwidth
- Input Impedance
- Isolation
- Etc.
Antenna Bandwidth (2)

Wideband antennas

Bandwidth expressed as a ratio

\[ \frac{f_{\text{max}}}{f_{\text{min}}} : 1 \]

E.g. 10:1 ⇒ Maximum frequency ten times greater than minimum frequency

Narrowband Antennas

Usually express as “fractional bandwidth,” or

\[ \frac{(f_{\text{max}} - f_c)}{f_c} \times 100 \quad \text{where} \quad f_c \approx \frac{f_{\text{max}} + f_{\text{min}}}{2} \]

E.g. 5% fractional bandwidth ⇒

5% deviation from center frequency can be tolerated
Summarizing

So far ...

Have characterized a single antenna (patterns, port characteristics)

But,

How do TX/RX antennas work together?
How do we use parameters to estimate gain of whole link?

Simplest Case: Free Space Propagation
Governed by Friis Transmission Equation

More complicated cases
Multipath, shadowing
Consider later in course (Propagation part of class!)
Friis Transmission Equation

\[
W_t = \frac{P_t G_t (\theta_t, \phi_t)}{4\pi R^2}
\]

- \(P_t\): Transmit power
- \(R\): Transmit/Receive separation
- \((\theta_t, \phi_t)\): Transmit direction
- \(G_t\): Transmit gain
Friis Transmission Equation (2)

Next, power collected by receive antenna
Need to derive notion of receiving aperture or “effective area”
Assuming antenna captures power from area $A_r$

$$P_r = \frac{P_t G_t A_r}{4\pi R^2}$$

Turns out that $A_r$ is a directional quantity related to $G_r$
Relation of Effective Area and Gain

Consider again (assumed)

$$P_r = W_t A_r = \frac{P_t G_t A_t}{4\pi R^2}$$

Now consider making 1=RX and 2=TX (switch roles)

Know by reciprocity that \textit{received power same}

$$P_r = \frac{P_t G_t A_t}{4\pi R^2}$$  \hspace{1cm} \text{(switched TX/RX)}

Comparing, this means that

$$G_t A_r = G_r A_t \quad \text{or} \quad \frac{G_t}{A_t} = \frac{G_r}{A_r}$$

Since we have specified nothing about antennas,

$$\frac{G}{A} = \text{constant} \quad \text{(ANY reciprocal antenna!)}$$
Friis Transmission Equation (3)

\[ \frac{G}{A} = \text{constant} \]

How do we find the constant?
Analyze a convenient antenna and find both \(G\) and \(A_r\)

Later we will do this for an infinitesimal (Hertzian) dipole:
\(A_r = \frac{3\lambda^2}{(8\pi)}\) and \(G_r = 1.5\)

Which means for any antenna
\(G_r/A_r = 4\pi/\lambda^2\)

\[
A_r(\theta, \phi) = \frac{G_r(\theta, \phi)\lambda^2}{4\pi}
\]

\[
P_r = \frac{P_t G_t(\theta_t, \phi_t)G_r(\theta_r, \phi_r)\lambda^2}{(4\pi R)^2}
\]
Summary

Standard terms and definitions for antennas

Radiation Parameters

Network (port) Parameters

Friis Transmission Equation

Next time: Start analyzing specific antenna types