Photonics and Optical Communication

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Review of optics

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Photonics and Optical Communication

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2.1 The nature of Light

Light propagation can be described by two mutually coupled vector waves, an electric-field wave and a magnetic field wave. In such a case we speak about *Electromagnetic Optics*. Electromagnetic optics allows us to describe most of the optical phenomena.

Nevertheless, it is possible to describe many optical phenomena using a scalar wave, in which light is described by a single scalar wave function. In such a case the electromagnetic wave is described by *Wave optics*.

When light propagates through and around objects whose dimensions are much greater than the wavelength its behavior can be adequately described by rays obeying geometrical rules. This model of light is called *Ray optics* (or *geometric optics*).

*Description of optical phenomena.*
2.1 The nature of Light

Although **electromagnetic optics** is the most complete description of light within the classical optics, there are certain optical phenomena which cannot be explained by a classical model. These phenomena are described by quantum electromagnetic theory. This model of light is called **quantum optics**.

Historically the optical theory was developed in the following sequence: (1) ray optics, (2) wave optics, (3) electromagnetic optics, (4) quantum optics. All models are progressively more difficult and complex.

The theory of quantum optics provides an explanation of virtually all optical phenomena. The electromagnetic theory of light provides the most complete treatment of light within the confined classical optics.
2.2 Ray optics

Ray optics is the simplest theory of light. Light is described by rays that travel in different optical media according to geometric rules. Ray optics is also called geometric optics.

2.2.1 The Refractive index

The index of refraction of a medium is the ratio of the velocity of light in vacuum \( c \) to the velocity of the light in the medium (phase velocity) \( v \):

\[
n = \frac{c}{v}
\]

Refractive index

With other words: It is the time taken by light to travel a distance \( d \) which equals \( d/v=nd/c \). It is thus proportional to the product \( nd \), known as the optical path length.
2.2.1 The Refractive index

An index of refraction is characterized by a value $n \geq 1$, called the refractive index. The refractive index of vacuum (air) is $n=1$.

The refractive index $n$ is a function of wavelength $\lambda$ (or frequency $f$) because the phase velocity $v$ depends on $\lambda$. The frequency of the wavelength of light are interrelated by $f = c/\lambda$.

Optical frequencies and wavelengths.

Ref: Saleh & Teich, Fundamentals of Photonics
2.2.2 Propagation of rays in homogeneous media

In a homogeneous media the refractive index is constant throughout the entire media. As a consequence the light travels at the same speed.

2.2.3 Reflection from a mirror

Mirrors are made of certain highly polished metallic surfaces or dielectric films deposited on a substrate such as glass. Light reflects in accordance with the law of reflection.

**Law of reflection**: The reflected ray lies in the plane of incidence; the angle of reflection equals the angle of incidence.

Reflection from a curved surface.

Ref: Saleh & Teich,

Fundamentals of Photonics
2.2.4 Snell’s laws of reflection and refraction

At the boundary between two media of refractive indices $n_1$ and $n_2$ an incident ray is split into two – a reflected ray and a transmitted ray. The reflected ray obeys the law of reflection. The transmitted ray obeys the law of refraction.

The transmitted ray lies in the plane of incident; the angle of transmission $\theta_2$ is related to the angle of incidence $\theta_1$ by Snell’s law.

$$n_1 \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$$

Snell’s law.
2.2.4 Snell’s laws of reflection and refraction

The transmitted ray lies in the plane of incident; the angle of transmission $\theta_2$ is related to the angle of incidence $\theta_1$ by Snell's law.

With increasing angle of incidence $\theta_1$ the angle of refraction $\theta_2$ also increases. If $n_1 > n_2$, there comes a point when $\theta_2 = \pi/2$ radians. This happens when $\theta_1 = \sin^{-1}(n_2 / n_1)$. For larger values of $\theta_1$, there is no refracted ray, and all then energy from the incident ray is reflected.

Reflection of rays at an interface for different incident angles, (a) From high to low refractive media, (b) For an critical angle, (c) Total internal reflection

Ref: J.M. Senior, Optical Fiber Communication
2.2.5 Total internal reflection

This phenomena is called **total internal reflection**. The smallest angle we get total internal reflection is called the **critical angle** and $\theta_2$ equals $\pi/2$ radians.

\[
\sin \theta_c = \frac{n_2}{n_1}
\]

The total internal reflection is an requirement for the guidance of light in an optical fiber.

Transmission of a light ray in a perfect optical fiber.

Ref: J.M. Senior, Optical Fiber Communication
2.3 Wave optics

The classical question typically raised: „Is light a particle phenomena or a wave phenomena?“

The question is far from being simple to answer.

So far we described light as a ray. Ray optics can be described by the particle phenomena of light. The essential feature of a particle is that a particle has a location. Practically we assume to be a particle something like a ball or a pebble with a very small diameter. But a particle interacts with its environment via gravitation, because each particle has a mass. The gravitation field extends into space. Furthermore, the gravitation field cannot be separated from the gravitation field. The gravitation field does not exist without the particle and the particle does not exist without the gravitation field.

However, photons (the smallest) do not behave like ordinary particles.
2.3 Wave optics

An understanding of the nature of light is essential to a complete description of optics.

The wave theory of light encompasses the ray theory. Ray optics is the limit of wave optics when the wavelength is infinitesimally short.

In this chapter, light is described by a scalar function, called the wave function, which obeys the wave equation.

Description of optical phenomena.
2.3.1 Longitudinal and transverse Waves

The essential feature of a wave is its non-localization. A classical traveling wave is a self-sustaining disturbance of a medium, which moves through space transporting energy and momentum.

In real life we are most familiar with mechanical waves like the waves of strings, surfaces waves on liquids and sound waves in air.

Sounds waves are **longitudinal waves**, where the media is displaced in the direction of the motion of the wave.

Waves on a string are **transverse wave**, where the medium is displaced in a direction perpendicular to that of the motion of the wave.

(a) Longitudinal wave in a spring, (b) Transverse wave in a spring, Ref.; E. Hecht, Optics
2.3.1 Longitudinal and transverse Waves

For both kinds of waves the energy carrying disturbance is advancing through the media. The atoms of the media remain at their position. Therefore, the disturbance advances and not the material medium.

2.3.2 The wave equation

An optical wave can be described mathematically by a real function of the position \( r=(x,y,z) \) and the time \( t \), denoted \( u(r,t) \) and is known as a wave function. The wave function has to satisfy the wave equation:

\[
\nabla^2 u - \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2} = 0
\]

\( \nabla^2 \) is the Laplacian operator \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \). The wave function can be derived from the Maxwell equations. The wave function is linear. Therefore, the principle of superposition applies.
2.3.2 The wave equation

The simplest possible wave can be described by a sine or a cosine function. These waves are known as **harmonic waves**. A harmonic (monochromatic) waves are represented by wave functions with harmonic time dependence. Monochromatic light is light consisting of a single wavelength.

\[
u(r, t) = a(r) \cdot \cos \left[ 2\pi ft + \phi(r) \right]
\]

Harmonic wave

\[
a(r) = \text{amplitude} \quad f = \text{frequency (Hz)}
\]

\[
\phi(r) = \text{phase}
\]

Both the amplitude and the phase are position dependent, but the wave function is a harmonic function of time with frequency at all positions.
2.3.2 The wave equation

Instead of using a simple real function \( u(\mathbf{r},t) \) the wave function can be described by a complex wave function \( U(\mathbf{r},t) \). The complex wave function can be described by:

\[
U(\mathbf{r},t) = a(\mathbf{r}) \cdot \exp \left( j \varphi(\mathbf{r}) \right) \cdot \exp \left( j 2\pi ft \right)
\]

Harmonic wave

After using the Euler formula

\[
\exp(j\theta) = \cos \theta + j \sin \theta
\]

Euler formula

The complex wave equation is given by

\[
\nabla^2 U - \frac{1}{c^2} \cdot \frac{\partial^2 U}{\partial t^2} = 0
\]

Complex wave equation

The real and the complex wave equation have the same form.
2.3.2 The wave equation

Real and Complex Representation of the wave function

\[ \omega = 2\pi f = \frac{2\pi}{T} \]

Angular frequency

Representation of a monochromatic wave at a fixed position \( r \):

(a) The real wave function \( u(r) \) is a harmonic function of time,

(b) The complex amplitude \( U = a \cdot \exp(j\varphi) \),

(c) Complex wave function \( U(t) = U \cdot \exp(j2\pi ft) \).

Ref: Saleh & Teich, Fundamentals of Photonics
2.3.2 The wave equation

The real wave function is now given by the real part of the complex wave function

\[ u(\vec{r}, t) = \text{Re} \left\{ U(\vec{r}, t) \right\} \]

And amplitude and the phase of the complex wave function is given by the absolute value and the argument of the complex wave function

\[ a(\vec{r}) = |U(\vec{r}, t)| \]
\[ \phi(\vec{r}) = \arg\left\{ U(\vec{r}, t) \right\} \]

The complex wave function can rewritten in the following form

\[ U(\vec{r}, t) = a(\vec{r}) \cdot \exp(j\phi(\vec{r})) \cdot \exp(j2\pi ft) = U(\vec{r}) \cdot \exp(j2\pi ft) \]
2.3.3 The Helmholtz equation

Leading to the following expression for the wave function

\[ U(\vec{r}, t) = U(\vec{r}) \cdot \exp(j2\pi ft) = U(\vec{r}) \cdot \exp(j\omega t) \]

In this case the amplitude is represented by a complex amplitude. If we now substitute the complex wave function by the new expression for the wave function we will get the following equation which is called the Helmholtz equation,

\[
\left( \nabla^2 + k^2 \right) U(\vec{r}) = 0
\]

Helmholtz equation

where the constant \( k \) is the wavenumber, which is defined as

\[
k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}
\]

Wavenumber
2.3.3 The Helmholtz equation

The complex wave function of light in the visible part of the spectrum can not be measured directly. The intensity can be calculated by the square of the complex wave amplitude

\[ I(\vec{r}) = |U(\vec{r})|^2 \]

Intensity of a wave
2.3.4 Summary Wave optics

Wave equation, Monochromatic light, Complex Representation of waves

A monochromatic wave of the frequency f can be described by a complex wave function $U(r,t)=U(r) \exp(j2\pi ft)$, which satisfies the complex wave equation.

The wave function $u(r,t)$ is the real part of the complex wave function $U(r,t)$.

The complex amplitude $U(r)$ satisfies the Helmholtz equation; its magnitude $|U(r)|$ and argument $\arg(U(r))$ are the amplitude and the phase of the wave. The optical intensity is equal to $I(r)=|U(r)|^2$.

As a harmonic wave (monochromatic wave) propagates through media of different refractive indices its frequency remains the same, but its velocity, wavelength and wavenumber is altered.

$$c_{\text{media}} = v = \frac{c}{n_{\text{media}}}$$

$$\lambda_{\text{media}} = \frac{\lambda}{n_{\text{media}}}$$

$$k_{\text{media}} = n_{\text{media}}k$$
2.4 Wave propagation

2.4.1 Plane waves

The plane wave is the simplest example of a three-dimensional wave. A plane wave exists when all waves have a **constant phase** and the surface forms a plane. The description of a wave as a plane wave is of high relevance as a lot of devices and optical phenomena can be described by plane waves.

\[
U(\vec{r}) = A \cdot \exp\left(- j \vec{k} \cdot \vec{r}\right) \\
= A \cdot \exp\left[- j(k_x x + k_y y + k_z z)\right]
\]

The plane wave is typically described by a vector \( \vec{k} = (k_x, k_y, k_z) \) (wavevector), which is perpendicular to the plane wave.

---

A plane wave moving in the \( \vec{k} \)-direction,

*Ref: E. Hecht, Optics*
2.4.1 Plane waves

The wave function of a plane wave can be described by

\[ U(\mathbf{r}) = A \cdot \exp(-j\mathbf{k} \cdot \mathbf{r}) \]

\[ = A \cdot \cos(\mathbf{k} \cdot \mathbf{r}) \]

\[ = -A \cdot \sin(\mathbf{k} \cdot \mathbf{r}) \]

The planes repeat themselves in space after a displacement of \( \lambda \) in the direction of \( \mathbf{k} \).

Wavefronts of a harmonic plane wave.

Ref: E. Hecht, Optics
2.4.2 Spherical waves

Another simple solution of the Helmholtz equation is a spherical wave, where \( r \) is the distance from the origin and \( k \) is the wavenumber. Spherical waves are concentric spheres separated by \( \lambda = \frac{2\pi}{k} \).

\[
U(\vec{r}) = \frac{A}{r} \cdot \exp\left( -jk \cdot \vec{r} \right)
\]

The intensity is given by \( I(r) = |A|^2/r^2 \).

Cross section of the wave function of a spherical wave,

Ref: Saleh & Teich, Fundamentals of Photonics
2.4.3 Plane, paraboloidal and Spherical waves

Besides plane and spherical waves other solutions for the Helmholtz equation can be found like paraboloidal waves.

All three descriptions of waves have a practical relevance. Waves can be described as spherical waves very close to the light source, further away the waves can be described as paraboloidal wave and very far away from the source the wave can be seen as a plane wave.

A spherical wave may be approximated at a point near the z axis and sufficiently far away from the origin by a paraboloidal wave. For points very far away from the origin the wave can be approximated by a plane wave.

Ref: Saleh & Teich, Fundamentals of Photonics
2.5 Interference

Interference cannot be explained on the basis of ray optics. Interference can only be explained by wave optics.

The interference requires the superposition of multiple (at least two) waves. Interference of two optical waves at the same time and position in space can be described by the superposition of the wave functions. The superposition is a consequence of the linearity of the wave equation. We already discussed that the superposition principle applies for the wave equation meaning the wave function.

The same is true for monochromatic light. The superposition principle applies for monochromatic light which is described by the Helmholtz equation.

The superposition does not apply to the intensity. The intensity of the two superimposed waves is not necessary the sum of the two intensities. The difference can be explained by the interference of the two waves.

In the following we will discuss the interference of two waves:

\[ U(\vec{r}) = U_1(\vec{r}) + U_2(\vec{r}) \]
### 2.5.1 Interference of two waves

The intensity of a complex wave function can be described by

\[
I_1 = \left| \mathbf{U}_1 \right|^2 \quad \text{and} \quad I_2 = \left| \mathbf{U}_2 \right|^2
\]

\[
\mathbf{U}_1 = \sqrt{I_1} \cdot \exp(j \varphi_1)
\]

\[
\mathbf{U}_2 = \sqrt{I_2} \cdot \exp(j \varphi_2)
\]

So that the overall intensity results to

\[
I = \left| \mathbf{U} \right|^2 = \left| \mathbf{U}_1 + \mathbf{U}_2 \right|^2 = \left| \mathbf{U}_1 \right|^2 + \left| \mathbf{U}_2 \right|^2 + \mathbf{U}_1^* \mathbf{U}_2 + \mathbf{U}_1 \mathbf{U}_2^*
\]

\[
I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\varphi_2 - \varphi_1)
\]

**Interference equation**

The intensity of the sum of two waves is not the sum of the intensities. An additional term has to be considered, which is attributed to the interference between the two waves. This term may be positive or negative corresponding to constructive and destructive interference.
2.5.1 Interference of two waves

Special case

\[ I_1 = I_2 = I_0 \]

leads to

\[ I = 4I_0 \cos^2 \left[ \frac{(\phi_2 - \phi_1)}{2} \right] \]

Phasor diagram for the superposition of two waves of the intensities \( I_1 \) and \( I_2 \) and the phase difference \( \phi = \phi_2 - \phi_1 \). Dependence of the total intensity \( I \) on the phase difference \( \phi \).

Ref: Saleh & Teich, Fundamentals of Photonics
2.5.1 Interference of two waves

Interference assumes that the superimposed waves are coherent. Coherence, however, cannot always be assumed. For example „ordinary light“ (e.g. sunlight or light from a light bulb) exhibits a random fluctuation of the phase. Therefore, the phase difference between the waves is given by random values, which are uniformly distributed between 0 and $2\pi$, so that the average of $\cos(\varphi)=0$ and the interference term disappears. As a consequence no interference is observed and the intensity of the superimposed waves is equal to the sum of the intensities.

Dependence of the intensity $I$ of the superposition of two waves. The two superimposed waves have the same intensity ($I_1=I_2=I_0$).

Ref: Saleh & Teich, Fundamentals of Photonics
2.5.1 Interference of two waves

The phenomena of interference is one of the most interesting and important aspects of optical engineering. Interference is the bases of almost everything we do in fiber optic communications.

The easiest way to study interference is the superposition of plane waves. The effect is usually applied as part of interferometers, where plane waves which propagate into opposite directions are superimposed. The superposition leads to the formation of an interference pattern, which can be detected.

An Mach Zehnder interferometers can be used as an optical switch.

Integrated optical intensity modulator (or optical switch) based on a Mach-Zehnder Interferometer. The optical switch is implemented as a wave guide structure in an electro optical material (e.g. LiNbO₃).

Ref: Saleh & Teich, Fundamentals of Photonics
2.5.2 Interference of plane waves

An alternative approach to superimpose two wave is the interference of two plane waves under an angle. The superposition of two plane waves under an angle leads to the formation of a sinusoidal intensity pattern on a screen. The intensities of the incident waves are the same.

Interference of two plane waves at an angle $\theta$ resulting in an sinusoidal intensity pattern on the screen. The spacing of the maxima of the intensity pattern is $\lambda / \sin(\theta)$.

Ref: Saleh & Teich, Fundamentals of Photonics
2.5.3 Interference of spherical waves (Young’s experiment)

In the second case two spherical waves of equal intensity interfere on a screen which is perpendicular to the y-plane. The experiment is similar to the Young’s experiment. The two spherical waves are formed by passing light through two pinholes.

The interference pattern is caused by the difference in distance traveled to the screen by light from the two slits. The waves interfere destructively or constructively depending on the phase condition.

*Young’s experiment: Superimposed spherical waves showing minima and maxima of the interference pattern.*

*Ref: E. Hecht, Optics*
2.5.3 Interference of spherical waves (Young’s experiment)

If the difference in distance traveled is an exact multiple of the wavelength we get constructive interference. Whereas, if the difference in distance is exact an odd multiple of the half wavelength we get destructive interference and a bright band.

In order to calculate the period of the intensity pattern on the screen we can carry out the following calculation:

\[ r_1 - r_2 = a \cdot \sin \Theta \]

since \( \sin \Theta \approx \Theta \)

\[ \Rightarrow r_1 - r_2 \approx a \cdot \Theta \]

\[ \Theta \approx \frac{y}{s} \]

Geometry of the Young’s experiment.

Ref: E. Hecht, Optics
2.5.3 Interference of spherical waves (Young’s experiment)

We can continue the calculation by

\[ r_1 - r_2 \approx \frac{a}{s} y \]
\[ r_1 - r_2 = m \cdot \lambda \]
\[ y_m \approx \frac{s}{a} m \cdot \lambda \]
\[ \Theta_m = m \frac{\lambda}{a} \]
\[ y_m \approx \frac{\lambda}{\Theta} \]

Geometry of the Young’s experiment.

Ref: E. Hecht, Optics
2.5.4 Temporal Coherence

So far we assumed that light is coherent. For example we used the assumption of coherence to induce planar and spherical waves.

Coherence is referring to the phase relationship of waves.

A wave is considered to be a coherent wave if the components of waves have a well defined phase relationship. A wave is incoherent if the components of the waves have a random phase relationships.

The theory of optical coherence deals with the definition of the statistical optics, by which light is classified into coherent, incoherent and partially coherent light.

Example of a coherent wave (a) and incoherent wave (b).

Ref: Agilent Technologies, Back to Basics in Optical Communications Technology
2.5.4 Temporal Coherence

Consider the fluctuation of stationary light at a fixed position \( r \) as a function of time. The stationary (average) random wave function has a constant intensity.

Is the lightwave for a fixed point in space fairly sinusoidal a certain time/period we can define the extend of the regular oscillation of the wave as coherence time and coherence length.

Coherence time (\( t_c \)): Average time for the wave train to lose its phase relationships
Coherence length (\( L_c \)): Average distance over which superposed waves lose their phase relationships. The coherence length can be calculated as the product of the coherence time times the speed of light.

\[
L_c = c t_c
\]

Relationship between coherence length and coherence time

It is convenient to describe the propagating light as a progression of a well-defined more or less sinusoidal waves which have an average length (coherence length) whose phase is correlated.
2.5.4 Temporal Coherence

If the light were ideally monochromatic the wave would be a perfect sinusoid with an infinite coherence length. All real sources are short of that and all sources emit a range of frequencies, which are in the case of a laser quite narrow. The coherence length and the coherence time are therefore related to the range of frequencies (spectral width) of a wave. With other words: The bandwidth determines the coherence length and time.

**Short coherence time ⇔ Broadband Source**

\[ \Delta v_c = \frac{1}{t_c} \]

Relationship between spectral width and coherence time

Coherence length, coherence time and spectral width of light sources

<table>
<thead>
<tr>
<th>Source</th>
<th>Spectral Width (Hz)</th>
<th>Coherence time</th>
<th>Coherence length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtered sunlight</td>
<td>$3.57 \times 10^{14}$</td>
<td>2.67fs</td>
<td>800nm</td>
</tr>
<tr>
<td>Light emitting diode</td>
<td>$1.5 \times 10^{13}$</td>
<td>67fs</td>
<td>20µm</td>
</tr>
<tr>
<td>Multimode HeNe-laser</td>
<td>$1.5 \times 10^{9}$</td>
<td>670ps</td>
<td>20cm</td>
</tr>
<tr>
<td>Single mode HeNe-laser</td>
<td>$1 \times 10^{6}$</td>
<td>1µs</td>
<td>300m</td>
</tr>
</tbody>
</table>
2.5.5 Spatial Coherence

So far we have studied the coherence of a wave in terms of temporal coherence, where we concentrated on a point and studied how the wave oscillates for a given position as a function of time. Moreover, the coherence of a wave front can be studied, where the time is constant and the coherence exists along the wavefront. In this case we speak about spatial coherence.
2.5.6 Transmission through a transparent plate

Now we will examine the transmission of optical waves through a transparent optical component like a glass plate, lenses or prisms. Here we will only discuss the transmission of light through a glass plate. So far we have not introduced absorption. Therefore, all light will be transmitted through the glass plate. Furthermore, we will ignore the reflection at the surfaces. The main emphasis here is on the phase shift introduced by these transparent optical components.

Consider the transmission of a plane wave $\mathbf{U}(\mathbf{r},t)$ through a transparent plate with a refractive index $n$ and a thickness $d$. The glass plate is in the plane $z$ and the wave travels in the direction of $z$.

Transmittion of a plane wave through a transparent plate.

Ref: Saleh & Teich, Fundamentals of Photonics
2.5.6 Transmission through a transparent plate

The transmission of light through a glass plate can be described by

\[ T(x, y) = \frac{U(x, y, d)}{U(x, y, 0)} \]

\[ T(x, y) = \exp(-jnkd) = \exp\left(-j2\pi \frac{nd}{\lambda}\right) \]

where \( t(x,y) \) is the transmission of the plate. The plate leads to a phase shift of \( nkd = 2\pi (d/\lambda) \).

In the following we will include the reflection at the interfaces in our discussion. Let’s assume that the incident light is normal (or perpendicular) to the glass surface. Due to Snell’s law 4% of the light is reflected. (The percentage of reflected light is higher if the light is incident at a more oblique angle.)

The amount of that reflection depends on the difference in refractive index of the two materials.
2.5.6 Transmission through a transparent plate

As the amount of reflected light depends on the difference in refractive index of the materials we get the same 4% regardless of whether the light is coming from the air side or from the glass side.

Reflection of light at the glass/air interface

Ref.: Harry Dutton, Understanding Optical Communications
2.5.6 Transmission through a transparent plate

It now seems obvious to conclude that 92% of the light is transmitted, because 4% of the light is reflected at the air/glass and another 4% of the light is reflected at the glass/air interface \( t_{\text{air/glass}} \times t_{\text{glass/air}} = (t_{\text{air/glass}})^2 \). Surprisingly this is not the case.

*Interference effect within a glass sheet.*

*Ref.: Harry Dutton, Understanding Optical Communications*
2.5.6 Transmission through a transparent plate

Interestingly the reflection depends on the thickness of the glass sheet:

- With increasing thickness more light is reflected up to a point where 16% of the light is reflected.
- If the thickness increased further the reflection will decrease down to zero.
- As we continue the reflection will again increase up to 16%.

Variation of the reflection of a glass sheet as a function of the thickness of the glass sheet. Ref: Harry Dutton, Understanding Optical Communications
2.5.6 Transmission through a transparent plate

The reflection is maximized (16%) if the round trip distance in the glass is a multiple of the wavelength. Therefore, the glass has have a thickness of a multiple of half of the wavelength \(d=m\times\frac{\lambda}{2}/n\) so that the round trip distance is multiple of the wavelength. Under such conditions the reflected wave is in phase with the incident wave and we observe constructively interference. In phase means that the phase of the reflected light is shifted by 180°.

How can we now explain the reflection of 16% of the light? The optical power corresponds to the square of the strength of the electric field. What we are really adding is electric fields. So on a single surface the reflection is 20% of the electric field strength. Squaring of the electric field gives 4% of the optical power. If we add 20% from the front surface and 20% from the back surface and then square this we get 16%.

The minimum reflection (0%) occurs when the round trip distance in the glass is out of phase with the incident light. In this case the thickness of the glass is an odd number of a multiple of the quarter wavelength. Under such conditions the reflected wave is out of phase, which leads to destructive interference. 100% of the incident light now passes through the glass and can be measured on the other side!
2.5.6 Transmission through a transparent plate

The most obvious use of this principle of destructive interference is in “anti-reflection coatings”. If you coat the surface of a piece of glass (or the end of a fiber) with a quarter-wavelength thick coating of a material of refractive index intermediate between that of the glass and that of air, reflections are significantly reduced. This is used in coating lenses of cameras, in solar cells and in many fiber-optic devices. This applies only for a single wavelength.

The described concept of destructive and constructive interference applies only for thin plates/films, where the thickness of the plate or the film is only a couple times the wavelength of the monochromatic light. If the plates gets ticker (like for normal window glass) the effect is typically channeled out.
2.6 Diffraction

When an optical wave is transmitted through an aperture and travels some distance in free space its intensity distribution starts to spread out and the intensity distribution is called diffraction pattern. If we would treat light as a ray the diffraction of light would be considered to be the shadow of the aperture. Due to the wave nature of light however, the diffraction pattern differs slightly or substantially from the shadow of the aperture depending on the distance of the aperture and the observation plane, the wavelength of light and the dimensions of the aperture.

In general diffraction can be described by the interference of wave fronts which propagates beyond an obstacle. There is not a significant physical distinction between interference and diffraction. It has somewhat become common to use the term interference, when only a few waves superimpose, whereas in the case of diffraction a large number of waves superimpose.

The description of diffraction theory is complex and far beyond the scope of this lecture/course.
2.6.1 Huygens‘s Principle

The most complete and powerful description of the theory of light is the Quantum Electro Dynamic (QED) theory. (Richard Feynman is considered as one of the „fathers“ of Quantum Electro Dynamics.) The theory of QED however is far too complicated and impractical for most of the problems. Therefore it seems more appropriate to concentrate simple formalisms.

Let’s assume we like to pass a wavefront through an optical system like a set of lenses. How can we determine the new wave front?

Huygens suggested more than 300 years ago to consider each point of a propagating wavefront as a source of a spherical secondary wavelet so that at a later time the wavefront is described by the wavelets.
2.6.1 Huygens‘s Principle

The frequency and the speed of the wavefront however remains unchanged.

At that time the idea of treating light as a wave was remarkable and provided new insights in optics.

Later on the ideas of Huygens were extended and modified by Fresnel. Fresnel for example introduced the concept of interference, which can not be described by Huygens concept.

*Huygen suggested to describe a propagating wave as if the wavefront were composed of point sources which emit spherical waves.*

*Ref: E. Hecht, Optics*
2.6.1 Huygen’s Principle

The Huygens-Fresnel states that every point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency). The amplitude of the optical field at any point is the superposition of all these wavelets (considering their amplitude and relative phase).

If the wavelength is large in comparison to the aperture, the waves will spread out at large angles into the region beyond the obstacle.

Diffraction through an aperture with a varying wavelength. Ref.: Ulaby, Fundamentals of Applied Electromagnetics,
2.6.2 Fraunhofer and Fresnel Diffraction

Imagine having a screen, which is placed behind a single slit or pinhole. The screen and the pinhole are aligned in parallel. In the following, we will vary the distance between the screen and the pinhole to study the diffraction pattern on the screen.

One of the assumptions is that the width of the slit is small in comparison to the wavelength of the incident light.

Is the screen very close to the aperture, the image of the aperture is projected on the screen. The projection is almost identical with the aperture and no spreading of the intensity pattern is observed.

If the screen is moved further away from the pinhole, increasingly more features (structure) can be observed. A formation of fringes can be recognized. The phenomena is known as **Fresnel or near-field diffraction** (pattern).

If we continue to move the screen further away from the slit or pinhole, a continuous change of the diffraction (interference pattern) can be observed. As part of the process, the pattern starts spreading out.
2.6.2 Fraunhofer and Fresnel Diffraction

At a very large distance from the pinhole/slit we see that the pattern is still spreading, but the shape of the pattern is unchanged. Under such conditions we speak about the **Fraunhofer or far-field diffraction**.

Diffraction from a slit of width $D=2a$. (a) Shaded area is the geometrical shadow of the aperture. The dashed line is the width of the Fraunhofer diffracted beam. (b) Diffraction pattern at four axial positions marked by the arrows. The shaded area represents the geometrical shadow of the slit. Ref: Saleh & Teich, *Fundamentals of Photonics*

$$N_F = \frac{a^2}{\lambda z}$$

Fraunhofer number
2.7 Dispersion

Two velocities describe the propagation of electromagnetic waves: the phase and the group velocity. We will first discuss the phase velocity.

2.7.1 Phase Velocity

The propagation of a wave can be described by the wave equation

\[ u(\vec{r}, t) = a(\vec{r}) \exp(-j(kz - \omega t)) \]

In terms of the phase we can consider

\[ kz - \omega t = \text{const}. \]

So that we get the following relationship for the movement of the phase

\[ z(t) = \frac{\omega t}{k} + \text{const}. \]
2.7.1 Phase Velocity

If we now calculate the velocity

$$\frac{dz}{dt} = \frac{\omega}{k} = v_p$$

and we consider that

$$v_p = \frac{1}{\sqrt{\mu \varepsilon}}$$

The following relationship can be defined:

$$n \equiv \frac{c}{v_p}$$

Or it can written in the following form

$$n = \frac{\sqrt{\mu \varepsilon}}{\sqrt{\mu_0 \varepsilon_0}}$$

The phase velocity is determined by the speed of propagation of the crest of a wave. Ref.: Pollock & Lipson, Integrated Photonics
2.7.1 Group Velocity

The group velocity describes the speed of propagation of a light pulse or packets of light. Each light source has a certain spectral width so that

\[ \omega_1 = \omega + \Delta \omega \quad \omega_2 = \omega - \Delta \omega \]

accordingly we get the associated wave vectors

\[ k_1 = k + \Delta k \quad k_2 = k - \Delta k \]

The group velocity can be then calculated by

\[ v_g = \frac{dz}{dt} = \frac{\Delta \omega}{\Delta k} \]

So that we get

\[ v_g = \frac{d\omega}{dk} = \frac{d}{dk} \left( \frac{kc}{n} \right) = \frac{c}{n} - \frac{kc}{n^2} \cdot \frac{dn}{dk} \]

Finally leading to

\[ v_g = \frac{c}{n} - \lambda \cdot \frac{dn}{d\lambda} \]

The propagation of light pulses leads to the formation of an envelope which propagates at the group velocity. Ref.: Pollock & Lipson, Integrated Photonics
2.7.1 Group Velocity

The group velocity is nearly equal to the phase velocity, but is reduced or increased by a small term proportional to the change of the refractive index with the wavelength. This change of the refractive index as a function of the wavelength is called dispersion.

**Regular dispersion:**

If $\lambda_1 > \lambda_2$, then $v_1 > v_2$ and therefore $n(\lambda_1) < n(\lambda_2)$

A way to remember this: “Red cars (e.g. Ferrari..) go faster than blue cars”

**Anomalous dispersion:**

If $\lambda_1 > \lambda_2$, then $v_1 < v_2$ and therefore $n(\lambda_1) > n(\lambda_2)$

Usually anomalous dispersion occurs only very close to the frequency of a transition between energy levels in the medium.

Dispersion plays an extremely important role when it comes to the propagation of light in a fiber!
2.8 Electromagnetic Optics

Light propagation can be described by two mutually coupled vectors waves, an electric-field wave and a magnetic field wave. In such a case we speak about *Electromagnetic Optics*.

In the following a brief review of the electromagnetic theory including the Maxwell equations will be given. Here we will concentrate on a more „intuitive“ description rather than a strict mathematical description of the underlying mathematics.

This hold true even though the Maxwell equations are one of the most „beautiful“ sets of equations in science. Maxwell can be definitely considered to be one of the greatest thinkers of all times.

*Description of optical phenomena.*
2.8.1 Electromagnetic theory of light

The electromagnetic field is described by two related vectors fields: the electric field $\mathbf{E}(r,t)$ and the magnetic field $\mathbf{H}(r,t)$. Both are vector functions of position and time. These two functions are related since they must satisfy a set of coupled partial differential equations known as Maxwell’s equations.

Maxwell’s equation in Free Space

The electric and magnetic fields in free space satisfy the following partial differential equation.

\[
\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

\[
\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}
\]

\[
\nabla \cdot \mathbf{E} = 0
\]

\[
\nabla \cdot \mathbf{H} = 0
\]

\[
\varepsilon_0: \text{ Electric permittivity,}
\]

\[
\mu_0: \text{ Magnetic permeability}
\]

Laplacian operator:

\[
\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}
\]

\[
\nabla \times \mathbf{E} = \begin{pmatrix}
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}
\end{pmatrix}
\]
### 2.8.1 Electromagnetic theory of light

Based on the first and the second Maxwell equation it is clear that an interdependence relationship between the electric field vector $\mathbf{E}$ and the magnetic field vector $\mathbf{B}$ exists. More precise:

Varying the electric field $\mathbf{E}$ generates a magnetic field $\mathbf{B}$, which is everywhere perpendicular to the direction in which the electric field changes. In the same way, a time-varying $\mathbf{B}$-field generates an $\mathbf{E}$-field, which is everywhere perpendicular to the direction in which $\mathbf{B}$ changes. As a consequence of this interdependence relationship between the $\mathbf{E}$-field and the $\mathbf{B}$-field we get a transverse electromagnetic wave.

This relationship can be illustrated by simply thinking of an electrical charge. Let's assume a charge being motionless in space. The resulting electric field is radial and constant over time. As a consequence there will be no magnetic field caused by the electric field. If now the charge starts moving the electric field in the vicinity of the charge starts to alternate and the alteration propagates into space. The time-varying electric field induces a magnetic field, which can be described by the second Maxwell equation.
2.8.1 Electromagnetic theory of light

If the charge’s velocity is constant, the rate-of-change of the \( E \)-field is steady and the resulting \( B \)-field is constant. If the charge is accelerated the derivation of the electric field is not constant and therefore the \( B \)-field is not constant. The time-varying \( B \)-field again generates an \( E \)-field and the process continues with \( E \) and \( B \) coupled (interdependence). As one field changes, it generates a new field that extends a little further into space so that the electromagnetic wave propagates from one point to another point. Therefore, the two fields – meaning the \( E \)-field and the \( B \)-field - are appropriately described by a single physical phenomena the electromagnetic field, whose source is a moving charge.

*Interdependence of the time varying \( E \)-field and \( B \)-field. The medium is displaced in a direction perpendicular to the motion of the wave (transverse wave).*

*Ref: Eugene Hecht, Optics*
2.8.1 Electromagnetic theory of light

Both fields are bound together as a single entity, the time-varying electric field and the time-varying magnetic field.

So far we have not discussed the direction of propagation of the electromagnetic wave. However, the high degree of symmetry in the Maxwell equations suggests that the disturbance will propagate in the direction that is symmetrical to both the $E$ and the $B$ field.

The first and the second Maxwell equation can be rewritten in the following form:

$$\nabla^2 \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Maxwell equations in a differential form

$$\nabla^2 \vec{B} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Note: The first two Maxwell equation can be rewritten by a total of 6 equations.
2.8.1 Electromagnetic theory of light

If we now compare the Maxwell equations with the wave equation we see that this form of equation has already been studied long before Maxwell’s work. The Maxwell’s equations in its differential form and the wave equation have the same form.

\[ \nabla^2 u = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2} \]

Wave equation

If we correlate the constants / prefactors of the two equations we get the following expression for the propagation speed of an electromagnetic wave in vacuum:

\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]

Speed of a light in a vacuum
2.8.2 Transverse Waves

How do we now understand the transverse character of the electromagnetic waves in terms of the Maxwell equations? It can be simply explained by assuming a planar wave in vacuum which propagates in the positive $x$-direction. Now the electric field has to satisfy the Maxwell equations (1. Maxwell equation in $x$-coordinates).

Furthermore, the electric field has to satisfy the third Maxwell equation, which states that the divergence of the electric field is zero in free space.

$$\frac{\partial E_x}{\partial x} = 0$$

The divergence of the electric field in $x$-direction can be zero if the electric field itself is constant. This tells us that the electric field is constant for all $x$ (electric field is not time-dependent). As the electric field is constant in $x$-direction we cannot speak of a wave traveling in $x$-direction caused by an electric field in $x$-direction. Therefore, the electric field in $x$-direction is zero, which means that the wave is not a longitudinal wave.
2.8.2 Transverse Waves

Propagation of an electromagnetic wave in x-direction. The fact that the E-field is transverse means that we will have to specify the moment-by-moment direction of the electric field. Such a description corresponds to the polarization of light (which will be discussed later). We can speak of linear polarized light (linear polarized waves), where the direction of the vibration of the E-field is fixed. The alternative would be circularly polarized light, where the direction of the vibration rotates along the direction of propagation.

Electromagnetic field propagating in x-direction. Orthogonal harmonic $E$- and $B$-fields for a planar wave. The wave propagates in $E \times B$ direction.

Ref: Eugene Hecht, Optics
2.8.3 Waves at interfaces

The electromagnetic theory leads to certain requirements in terms of electric fields at the interfaces (boundary conditions). One of these is that the tangential component of the electric field (and the magnetic field) on both sides of the interface must be continuous. With other words: The tangential component of the electric field on one side is equal to the electric field on the other side of the interface. The surface normal is described by $\mathbf{u}_n$.

$$\mathbf{u}_n \times \vec{E}_i + \mathbf{u}_n \times \vec{E}_r = \mathbf{u}_n \times \vec{E}_t$$

If we now assume that we can describe the electric field by:

$$\vec{E}_i = \vec{E}_{0i} \cos(k \cdot \mathbf{r})$$

We can derive Snell’s law:

$$n_i \sin \theta_i = n_t \sin \theta_t$$

Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

Ref: E. Hecht, Optics
2.8.4 Fresnel equations

We just found a relationship between the incident, the reflected and the transmitted electric field.

In the following we will distinguish between two cases:

(1) The incident electric field is perpendicular to the plane of incidence. As a consequence the magnetic field is parallel to the plane of incidence.

(2) In the second case the electric field is parallel to the plane of incidence. The magnetic field is therefore perpendicular to the plane of incidence.

(a) An incoming wave whose E-field is normal to the plane of incidence,

(b) An incoming wave whose E-field is in the plane of incidence,

Ref: E. Hecht, Optics
2.8.5 Transverse electric (TE) polarized wave

The subscript $\perp$ denotes that we are dealing with a case, where the electric field is perpendicular to the plane of incidence. We can call this wave a transverse electric (TE) polarized wave. In both cases we assume that the electric field is perpendicular to the plane of incidence. The Fresnel equations provide a general description which applies to linear, isotropic and homogenous media.

The parameter $r_\perp$ describes the **amplitude reflection coefficient** and $t_\perp$ the **amplitude transmission coefficient**.

\[
r_\perp \equiv \left( \frac{E_{0r}}{E_{0i}} \right)_\perp = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}
\]

\[
t_\perp \equiv \left( \frac{E_{0t}}{E_{0i}} \right)_\perp = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}
\]

An incoming wave whose E-field is normal to the plane of incidence,

Ref: E. Hecht, Optics
2.8.6 Transverse magnetic (TM) polarized wave

The subscript ‖ denotes that we are dealing with a case, where the electric field is parallel to the plane of incidence or we call the wave a transverse magnetic (TM) polarized wave. The electric field is parallel to the plane-of-incidence and the magnetic field is perpendicular to the plane of incidence. The Fresnel equations provide a general description which applies to linear, isotropic and homogenous media.

The parameter \( r_\parallel \) describes the amplitude reflection coefficient and \( t_\parallel \) the amplitude transmission coefficient.

\[
r_\parallel = \left( \frac{E_{0r}}{E_{0i}} \right)_\parallel = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i}
\]

\[
t_\parallel = \left( \frac{E_{0t}}{E_{0i}} \right)_\parallel = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \sin \theta_i}
\]

An incoming wave whose E-field is normal to the plane of incidence,

Ref: E. Hecht, Optics
2.8.7 Reflection

At nearly normal incidence ($\theta_i = 0$) the parallel and the perpendicular amplitude coefficients get identical. By combining the Fresnel equations with Snell’s law we get:

$$r_\parallel (\theta_i = 0) = -r_\perp (\theta_i = 0) = \frac{n_t - n_i}{n_t + n_i}$$

Amplitude reflection coefficient under normal incidence

We already used the equation when speaking about the reflection of a glass sheet. As the refractive indices are $n_{\text{air}} = 1$ and $n_{\text{glass}} = 1.45$ we get for the reflection 0.2.
2.8.8 Transmission

In terms of the transmission the Fresnel equations simplifies under normal incidence ($\theta_i=0$) to

$$t_{\parallel}(\theta_i = 0) = t_{\perp}(\theta_i = 0) = \frac{2n_i}{n_t + n_i}$$

Impendent of the angle of incidence the follow two equations have apply all time:

$$t_{\perp} - r_{\perp} = 1$$

Reflection and transmission coefficient under perpendicular incidence

$$t_{\parallel} + r_{\parallel} = 1$$

Reflection and transmission coefficient under parallel incidence
2.8.9 Amplitude Coefficients

When \( n_t > n_i \) it follows from Snell’s law that \( \theta_i > \theta_t \) and \( r_\perp \) gets negative for all angles. \( r_\parallel \) gets positive or equals zero when \( \theta_i + \theta_t = 90^\circ \). The particular value under which the parallel reflection is getting zero is called the polarization angle. Under such conditions the no electric field is reflected by the interface.

We can find a lot of examples in real life were exactly such conditions apply. For example at the interface between air and glass.

Amplitude coefficients of reflection and transmission as a function of the incident angle. The interface was described by an air / glass interface.

Ref: E. Hecht, Optics
2.8.9 Amplitude Coefficients

So far we discussed only the case of external reflection, where $n_t > n_i$. In the opposite case we speak about internal reflection, because $n_t < n_i$. Under such conditions $\theta_i < \theta_t$ and therefore $r_\perp$ will be always positive. The angle under which the amplitude coefficient gets 0 is called the critical angle. The term was already used when we introduced ray optics.

Amplitude coefficients of reflection and transmission as a function of the incident angle. The interface was described by an glass / air interface.

Ref: E. Hecht, Optics
2.8.9 Amplitude Coefficients

We can discuss the reflection and the transmission not only in terms of amplitudes. We can of course determine a phase for the different conditions. That gets clear when remind ourselves that the electric field is described by the vectors $E_{0i}$ and $E_{0r}$.

In general: The component of the electric field normal to the plane-of-incidence undergoes a phase shift of $180^\circ$ upon reflection when the incident medium has a lower index than the transmitting medium.

Phase shift for parallel and perpendicular components of the E-field corresponding to external reflection.

Ref: E. Hecht, Optics
2.8.9 Amplitude Coefficients

It gets a little bit more difficult to understand the case of internal reflection.

In general: Two fields in the incident plane are in-phase if their y-components are parallel and are out-of-phase if the components are antiparallel.

Phase shift for parallel and perpendicular components of the E-field corresponding to internal reflection.

Ref: E. Hecht, Optics
2.8.10 Reflectance and Transmission

So far we determined the amplitude reflection and transmission coefficients based on the reflected and transmitted electric and magnetic field. We already discussed before that we can only directly access the intensity of the light rather than the electric or the magnetic field. We can now define the reflectance of the intensity:

\[
R \equiv \frac{I_r A \cos \theta_r}{I_i A \cos \theta_i} = \frac{I_r}{I_i}
\]

\[
R = \left( \frac{E_{0r}}{E_{0i}} \right)^2 = r^2
\]

\[
R_{\perp} = r_{\perp}^2 \quad R_{\|} = r_{\|}^2
\]

*Reflectance and Transmission of an incident beam*

*Ref: E. Hecht, Optics*
2.8.10 Reflectance and Transmission

The procedure to define the transmission is very similar leading to:

\[ T \equiv \frac{l_t A \cos \theta_t}{l_i A \cos \theta_i} \]

\[ T = \frac{n_t \cos \theta_t \left( \frac{E_{0t}}{E_{0i}} \right)^2}{n_i \cos \theta_i} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t^2 \]

\[ T_{\perp} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t_{\perp}^2 \]

\[ T_{\parallel} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t_{\parallel}^2 \]
2.8.10 Reflectance and Transmission

So far we assumed that the medium is not absorbing light, therefore we can conclude that

\[ R + T = 1 \]

In general the same applied for the reflectance and the transmission which is perpendicular and parallel in terms of the electric field.

\[ R_\perp + T_\perp = 1 \quad R_\parallel + T_\parallel = 1 \]

If we furthermore assume that the angle of incidence is normal \((\theta_i = 0)\) we get:

\[
R = R_\perp = R_\parallel = \left( \frac{n_t - n_i}{n_t + n_i} \right)^2
\]

\[
T = T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}
\]

*Reflectance and Transmission versus incident angle.*

*Ref: E. Hecht, Optics*
2.9 Polarization

We already discussed in through the previous lectures that light may be treated as an transverse electromagnetic wave.

2.9.1 Linearly polarized light

However, so far we only considered that the electromagnetic wave was linearly polarized. Polarization means in this sense that the orientation of the electric field is constant, although the amplitude and the sign of the wave is alternating.

Ref: E. Hecht, Optics
2.9.2 Circular polarized light

Circular polarized light is an case or state of polarization which is of particular interest. Both, linearly and circular polarized light are special cases of elliptical polarized light.

Natural light:

Natural light is generally neither completely polarized nor completely unpolarized.

Ref: E. Hecht, Optics
References:


