ECMR 2007 – SLAM Tutorial
6D SLAM

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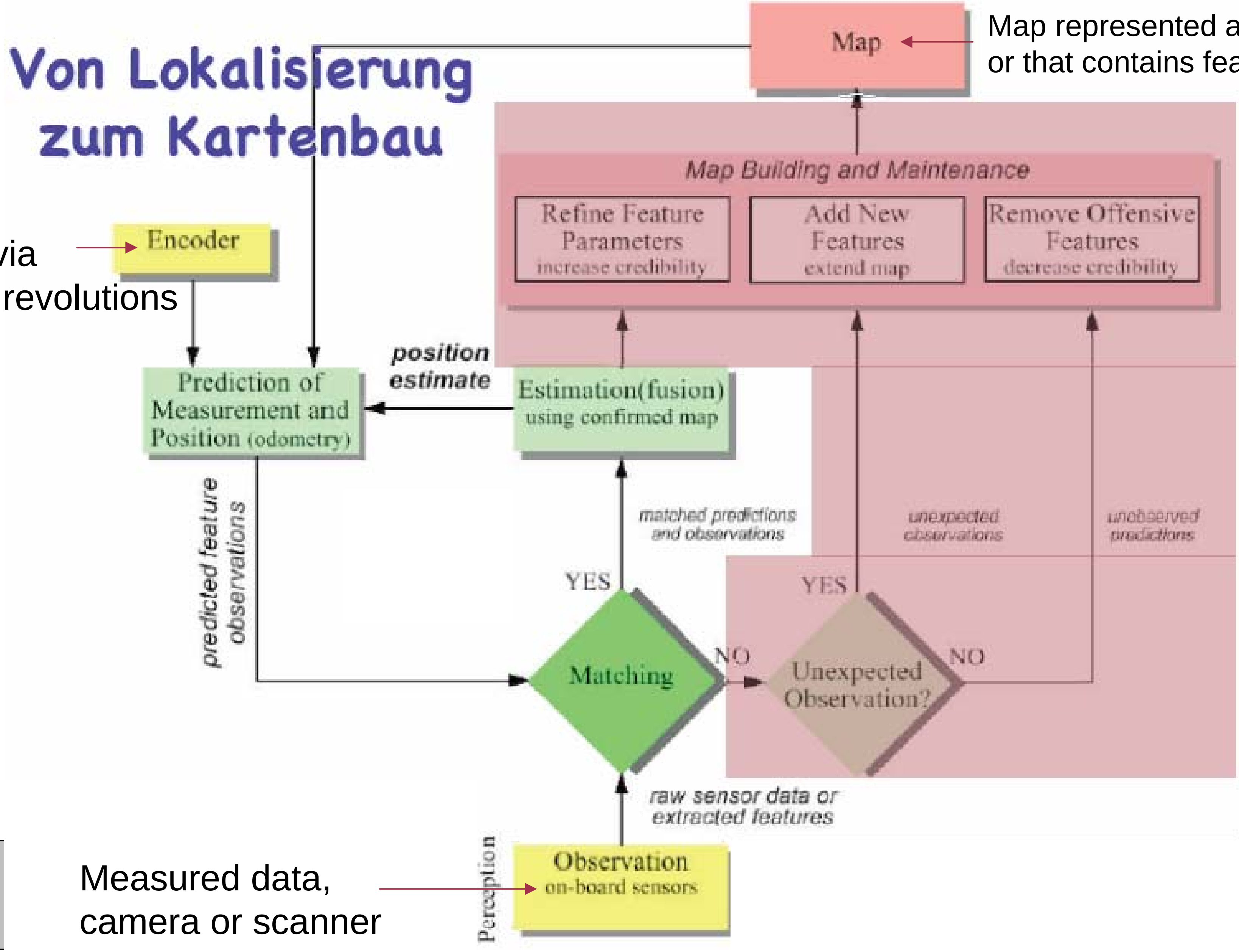
6D SLAM – Introduction (1)

- Up to now focus on
  - SLAM using a 2D grid map
  - Use of the particle filter
  - GraphSLAM
  - An efficient backend
- Now we will focus on pose estimates using **six degree of freedom**, thus **6D SLAM**
- We consider 3D laser scans as data
- Agenda

  1. Brief Introduction and Topic Statement
  2. Scan Matching
  3. Global Relaxation

6D SLAM – Introduction (2)



Map represented as grid or that contains features

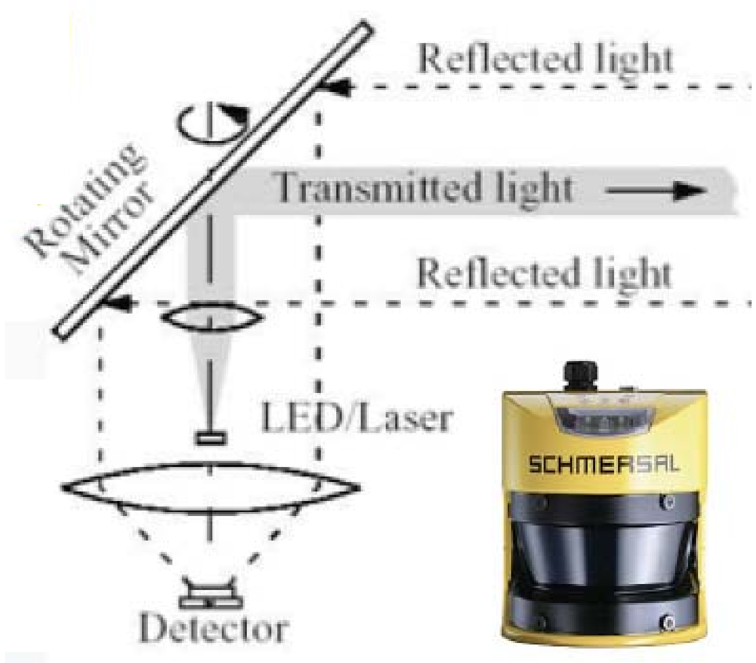
Pose via wheel revolutions

Measured data, camera or scanner

6D SLAM – Introduction (3)

c = 299.792.458 m/s (Vacuum), also

d = 299.792.458 [m/s] x *t*/2 (*d* Distance[m], *t* time-of-flight[s])

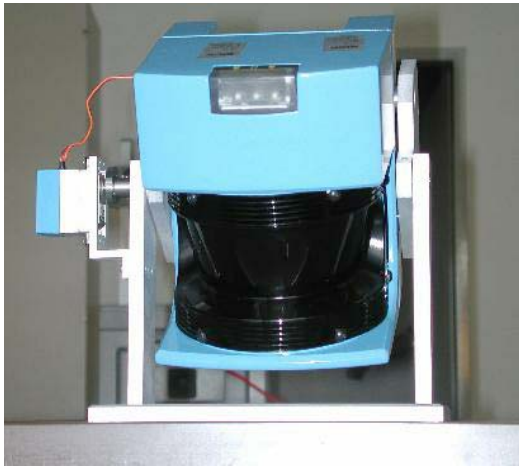


***c* ≈ 0,3 mm/ps**

➥ With a resolution of 10mm: Precision of the time-of-flight measurement in the order of pico seconds **(10^{-12} s)** needed!

6D SLAM – Introduction (4)

- 3D laser scanner for mobile robots based on SICK LMS



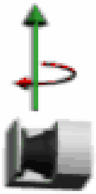
- Based on a regular (e.g., SICK LMS-200) laser scanner
- Relatively cheap sensor
- Controlled pitch motion (120° v)
- Various resolutions and modi, e.g., reflectance measurement {181, 361, 721} [h] x {128, …, 500} [v] points
- Fast measurement, e.g., 3.4 sec (181x256 points)

Mounted on mobile robots for 3D collision avoidance and building 3D maps.

(Video Crash)

(Video NoCrash)

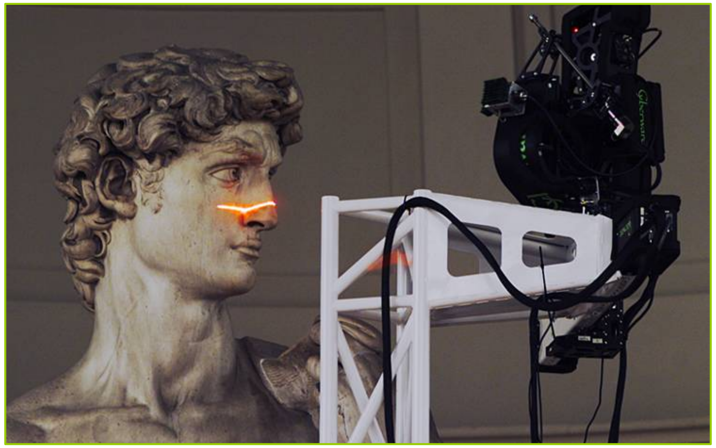
6D SLAM – Introduction (5)

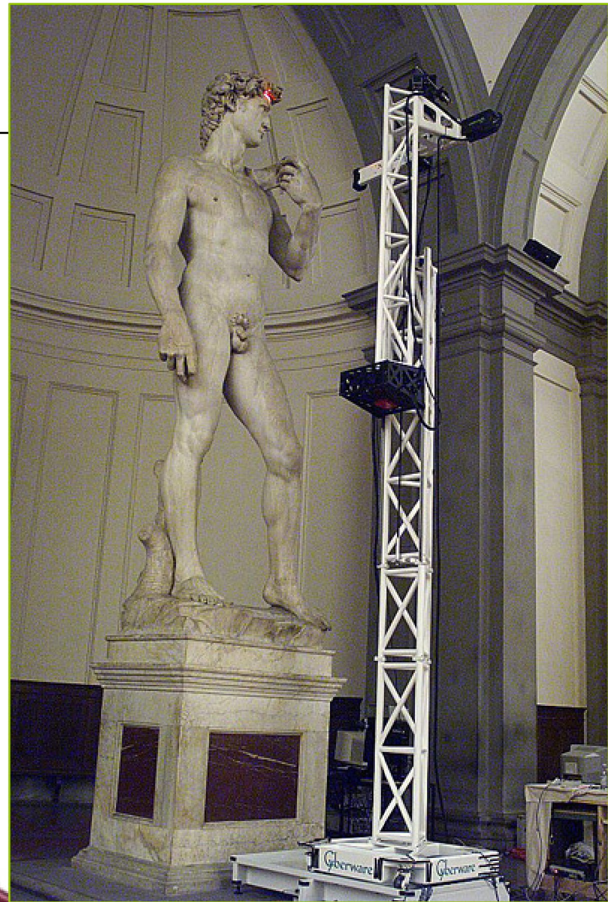
Mode	Symbol	Cont. rotating	pivoting	Advantages
Yaw				+ Complete 360° scans + Good point arrangements - High point density at top

http://www.rts.uni-hannover.de/index.php/%C3%9Cbersicht_der_m%C3%B6glichen_Scannerkonfigurationen

6D SLAM – Introduction (6)

- Professional 3D scanners
  - Structured light (close range)



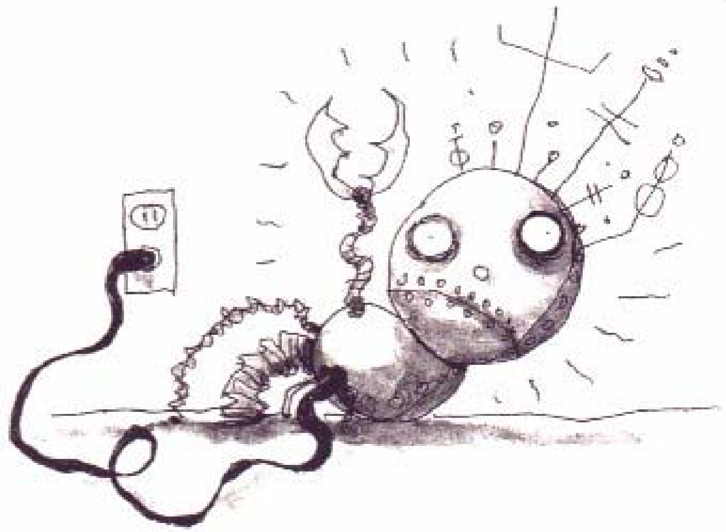


  - pulsed laser vs. time-of-flight (mid and long range)

Laserscanner
LEICA HDS3000

6D SLAM – Hands-on-experience (1)

- What you should learn now, using the `show` program
  - Most robotic data sets acquired by a rotating SICK scanner contain a lot of outliers
  - Data sets of professional scanners can be very large



- Things to try
  - Viewing a single outdoor data set

    `bin/show -s 0 -e 0 dat1`
  - Viewing a single indoor 3D scan, setting a maximal distance

    `bin/show -s 0 -e 0 -m 3000 dat2`
  - Viewing multiple indoor 3D scans

    `bin/show -s 0 -e 10 -m 3000 dat2`
  - Viewing a high resolution outdoor 3D scan

    `bin/showi -s 1 -e 1 dat_schillerplatz`

6D SLAM – Extrapolate Odometry in 6D

- Extrapolate the odometry readings to all six degrees of freedom using previous registration matrices

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \\ \theta_{x,n+1} \\ \theta_{y,n+1} \\ \theta_{z,n+1} \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \\ z_n \\ \theta_{x,n} \\ \theta_{y,n} \\ \theta_{z,n} \end{pmatrix} + \left(\begin{array}{c|c} \mathbf{R}(\theta_{x,n}, \theta_{y,n}, \theta_{z,n}) & \mathbf{0} \\ \hline \mathbf{0} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{array} \right) \cdot \underbrace{\begin{pmatrix} \Delta x_{n+1} \\ \Delta y_{n+1} \\ \Delta z_{n+1} \\ \Delta\theta_{x,n+1} \\ \Delta\theta_{y,n+1} \\ \Delta\theta_{z,n+1} \end{pmatrix}}_{\Delta P} .$$

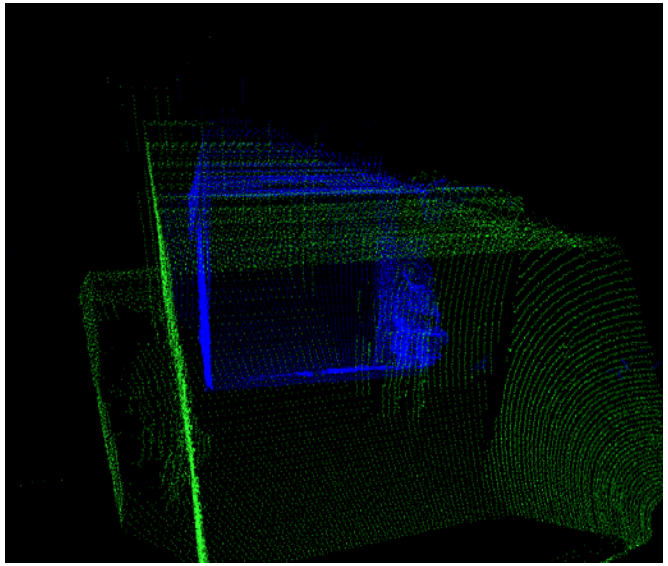
- Convert ΔP to matrix $\Delta \mathbf{P}$, then $\mathbf{P}_{n+1} = \Delta \mathbf{P} \cdot \mathbf{P}_n$

6D SLAM – The ICP Algorithm (1)

Scan registration Put two independent scans into one frame of reference

Iterative Closest Point algorithm [Besl/McKay 1992]

For prior point set M ("model set") and data set D



1. Select point correspondences $w_{i,j}$ in {0,1}
2. Minimize for rotation $\mathbf{R}$, translation $\mathbf{t}$

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m} \sum_{j=1}^{N_d} w_{i,j} \, ||\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})||^2$$

3. Iterate **1.** and **2.**

SVD-based calculation of rotation

- works in 3 translation plus 3 rotation dimensions
  $\Rightarrow$ 6D SLAM with closed loop detection and global relaxation.

6D SLAM – The ICP Algorithm (2)

Closed form (one-step) solution for minimizing of the error function

1. **Cancel the double sum:**

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})\right|\right|^2$$

$$\propto \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_i + \mathbf{t})\right|\right|^2,$$

2. **Compute centroids of the matching points**

$$\mathbf{c}_m = \frac{1}{N}\sum_{i=1}^{N} \mathbf{m}_i, \qquad \mathbf{c}_d = \frac{1}{N}\sum_{i=1}^{N} \mathbf{d}_j$$

$$M' = \{\mathbf{m}'_i = \mathbf{m}_i - \mathbf{c}_m\}_{1,\ldots,N}, \qquad D' = \{\mathbf{d}'_i = \mathbf{d}_i - \mathbf{c}_d\}_{1,\ldots,N}.$$

3. **Rewrite the error function**

$$E(\mathbf{R},\mathbf{t}) = \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i - \underbrace{(\mathbf{t} - \mathbf{c}_m + \mathbf{R}\mathbf{c}_d)}_{=\tilde{\mathbf{t}}}\right|\right|^2$$

6D SLAM – The ICP Algorithm (3)

Closed form (one-step) solution for minimizing of the error function

3. **Rewrite the error function**

$$\begin{aligned} E(\mathbf{R},\mathbf{t}) &= \frac{1}{N}\sum_{i=1}^{N}\left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i - \underbrace{(\mathbf{t} - \mathbf{c}_m + \mathbf{R}\mathbf{c}_d)}_{=\tilde{\mathbf{t}}}\right|\right|^2 \\ &= \frac{1}{N}\sum_{i=1}^{N}\left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i\right|\right|^2 - \frac{2}{N}\tilde{\mathbf{t}}\cdot\sum_{i=1}^{N}\left(\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i\right) + \frac{1}{N}\sum_{i=1}^{N}\left|\left|\tilde{\mathbf{t}}\right|\right|^2. \end{aligned}$$

- **Minimize only the first term! (The second is zero and the third has a minimum for $\tilde{\mathbf{t}} = 0$).**

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N}\left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i\right|\right|^2.$$

Arun, Huang und Blostein suggest a solution based on the singular value decomosition.

K. S. Arun, T. S. Huang, and S. D. Blostein. Least square fitting of two 3-d point sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5):698 – 700, 1987.

6D SLAM – The ICP Algorithm (4)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}',\ S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}',\ \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathrm{V}\mathrm{U}^T$ **with** $\mathbf{H} = \mathrm{U}\Lambda\mathrm{V}^T$ **from the SVD.**

Proof:

$$E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2 .$$

Rewrite

$$E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i'\right|\right|^2 - 2\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' + \sum_{i=1}^{N} \left|\left|\mathrm{d}_i'\right|\right|^2 .$$

Rotation is length preserving, i.e., maximize the term

$$\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' = \sum_{i=1}^{N} \mathrm{m}_{\mathrm{i}}'^T \mathrm{R}\mathrm{d}_i'$$

6D SLAM – The ICP Algorithm (5)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m'_{ix} d'_{ix},\ S_{xy} = \sum_{i=1}^{N} m'_{ix} d'_{iy},\ \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof: $\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' = \sum_{i=1}^{N} \mathrm{m}_\mathrm{i}'^T \mathrm{R}\mathrm{d}_i'$

Rewrite using the trace of a matrix

$$\mathrm{Trace}\left(\sum_{i=1}^{N} \mathrm{R}\mathrm{d}_i' \mathrm{m}_\mathrm{i}'^T\right) = \mathrm{Trace}\left(\mathrm{R}\mathbf{H}\right)$$

Lemma: For all positiv definite matrices $\mathbf{A}\mathbf{A}^T$ **and all orthonormal matrices** B **the following equation holds:** $\mathrm{Trace}\left(\mathbf{A}\mathbf{A}^T\right) \geq \mathrm{Trace}\left(\mathbf{B}\mathbf{A}\mathbf{A}^T\right)$

□

6D SLAM – The ICP Algorithm (6)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m'_{ix} d'_{ix}, \; S_{xy} = \sum_{i=1}^{N} m'_{ix} d'_{iy}, \; \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof: **Suppose the singular value decomposition of** H **is** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$

U **and** V **are orthonormal 3 x 3 and** Λ **a diagonal matrix without negative entries .**

$$\mathrm{R} = \mathbf{V}\mathbf{U}^T.$$

R **is orthonormal and**

$$\begin{aligned} \mathrm{RH} &= \mathbf{V}\mathbf{U}^T\mathbf{U}\Lambda\mathbf{V}^T \\ &= \mathbf{V}\Lambda\mathbf{V}^T \end{aligned}$$

And using the lemma it is $\mathrm{Trace}\,(\mathrm{RH}) \geq \mathrm{Trace}\,(\mathrm{BRH})\,.$

Therefore R **maximizes**

$$\sum_{i=1}^{N} \mathrm{m}_\mathrm{i}'^T \mathrm{R} \mathrm{d}_i'$$

□

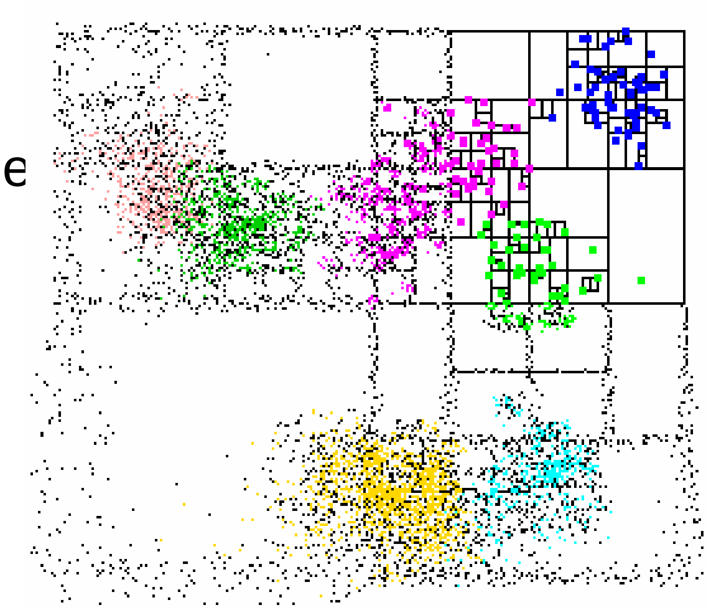
6D SLAM – The ICP Algorithm (7)

- Estimating the transformation can be accomplished very fast O(n)

- Closest point search
  - Naïve O(n^2), i.e., brute force
  - K-d trees for searching in logarithmic time

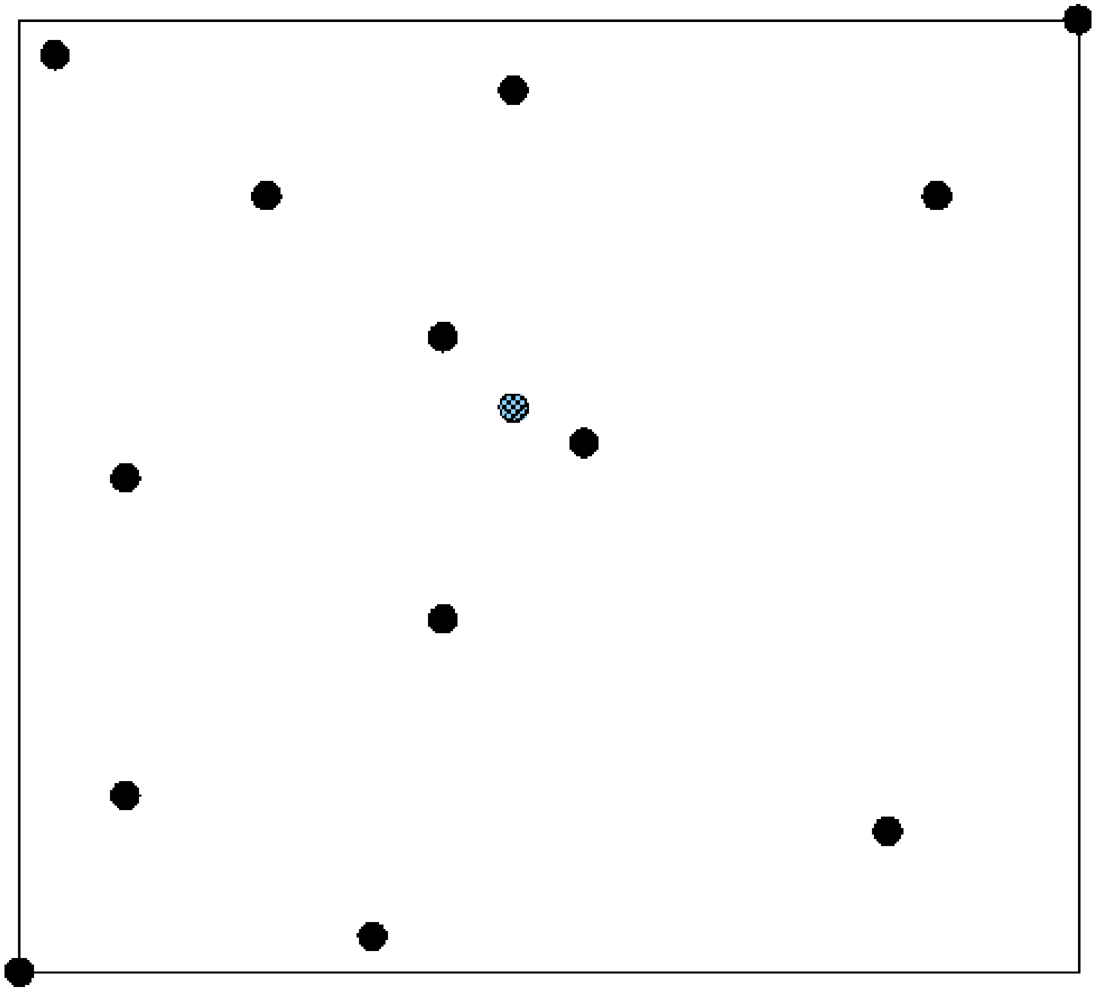
    Recommendation: Start with

    ANN: A Library for Approximate Nearest Neighbor Searching by David M. Mount and Sunil Arya (University of Maryland)
    - Easy to use
    - Many different methods are available
    - Quite fast

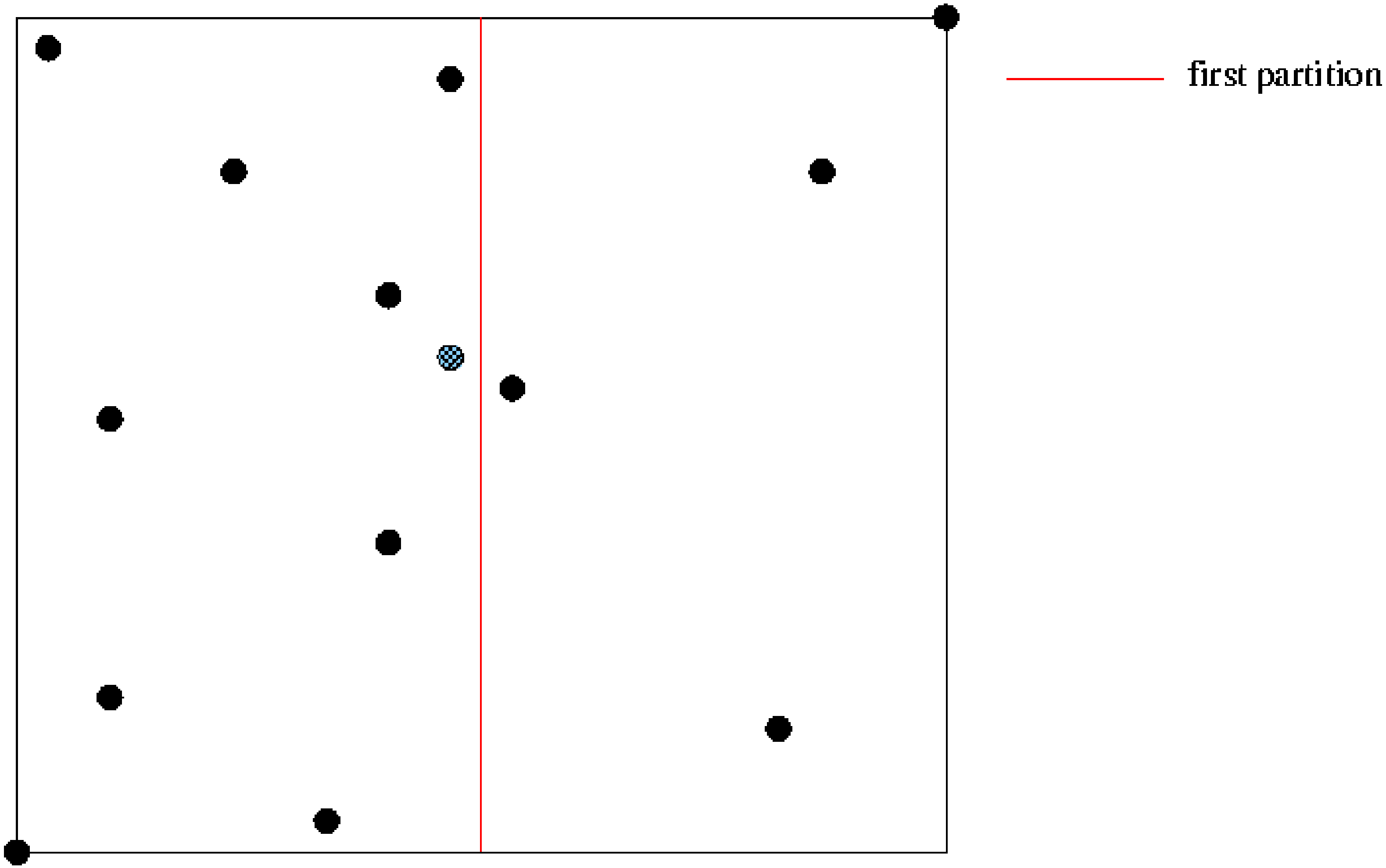


http://www.cs.umd.edu/~mount/ANN/

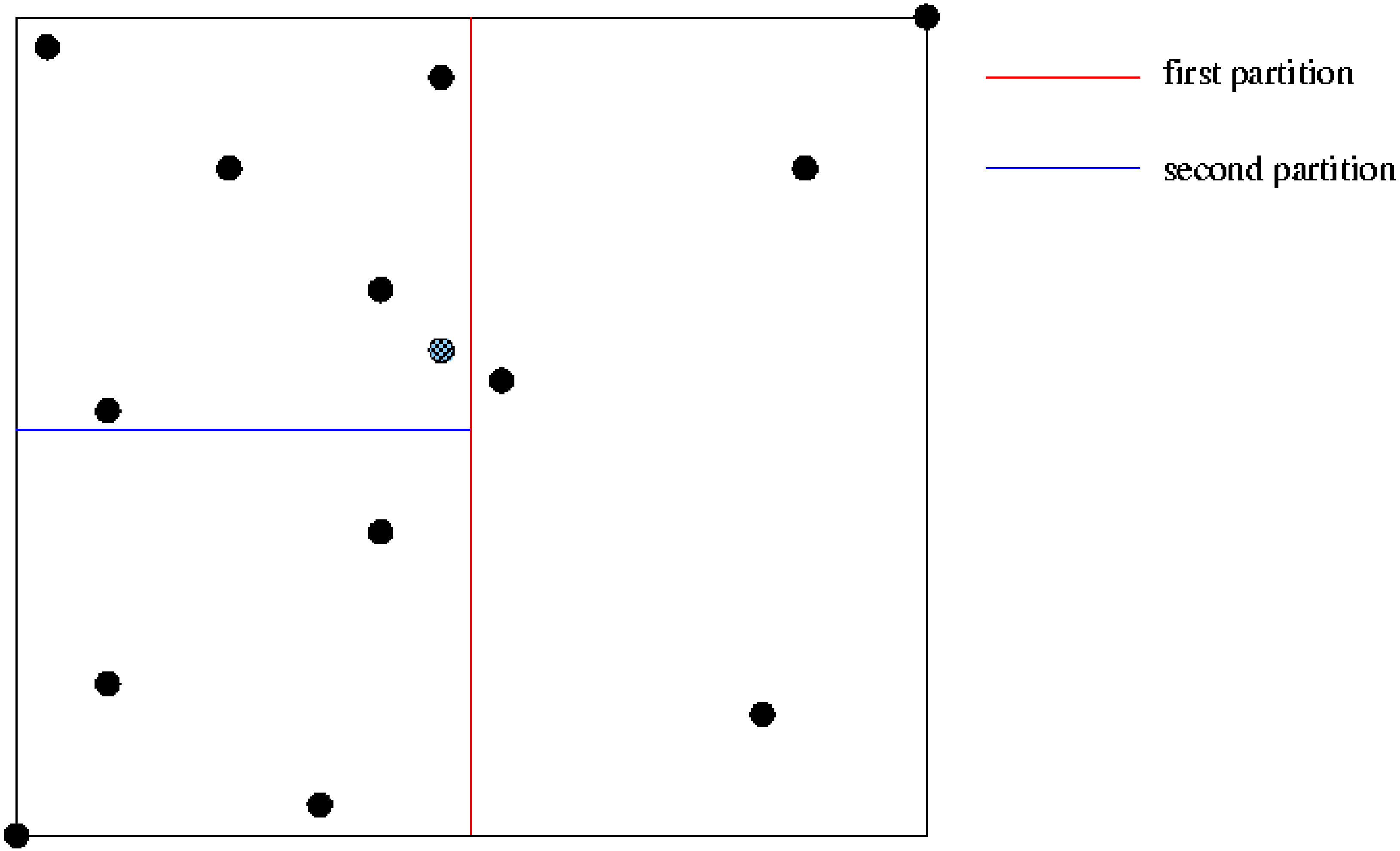
6D SLAM – The ICP Algorithm (8)



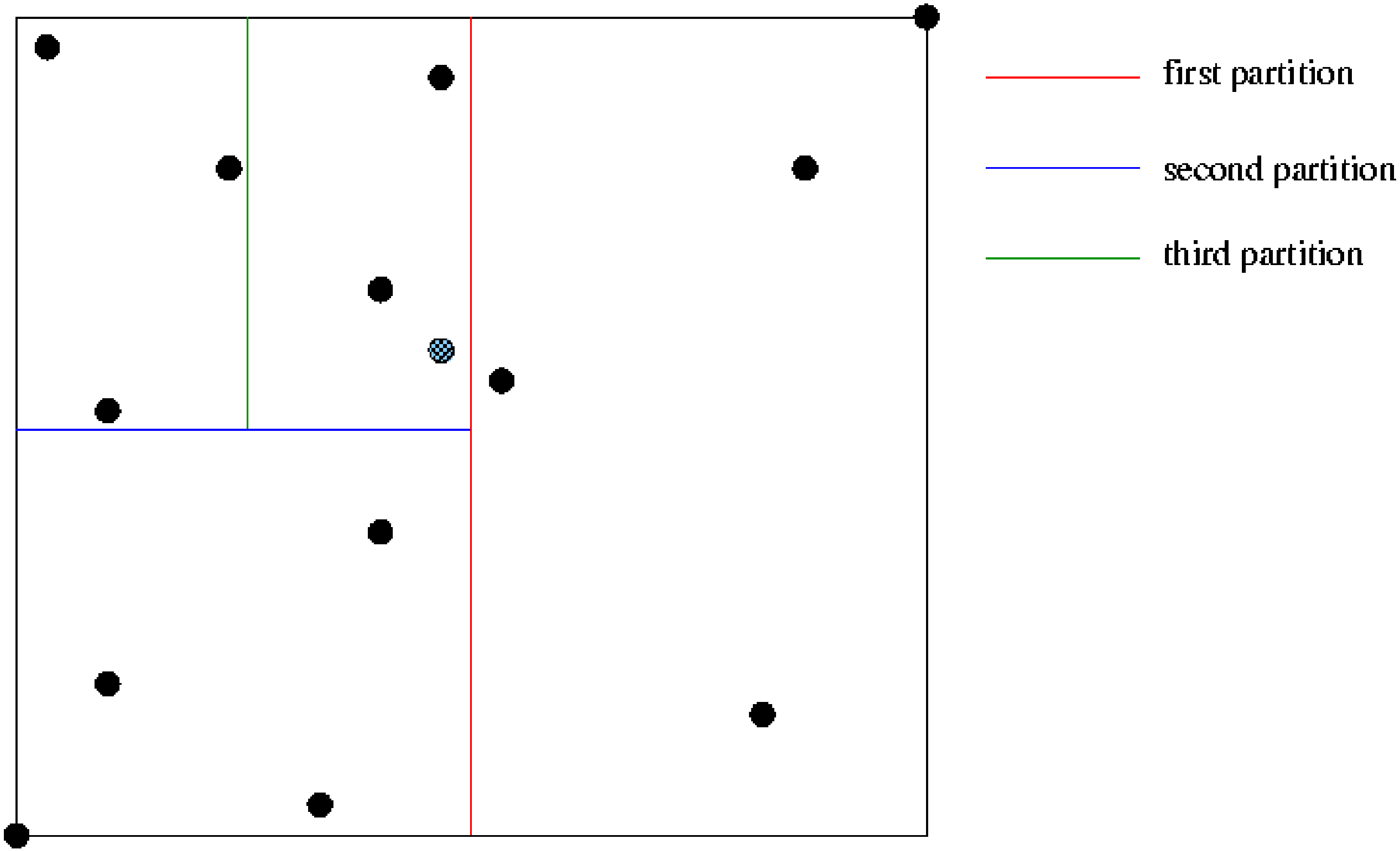
6D SLAM – The ICP Algorithm (9)



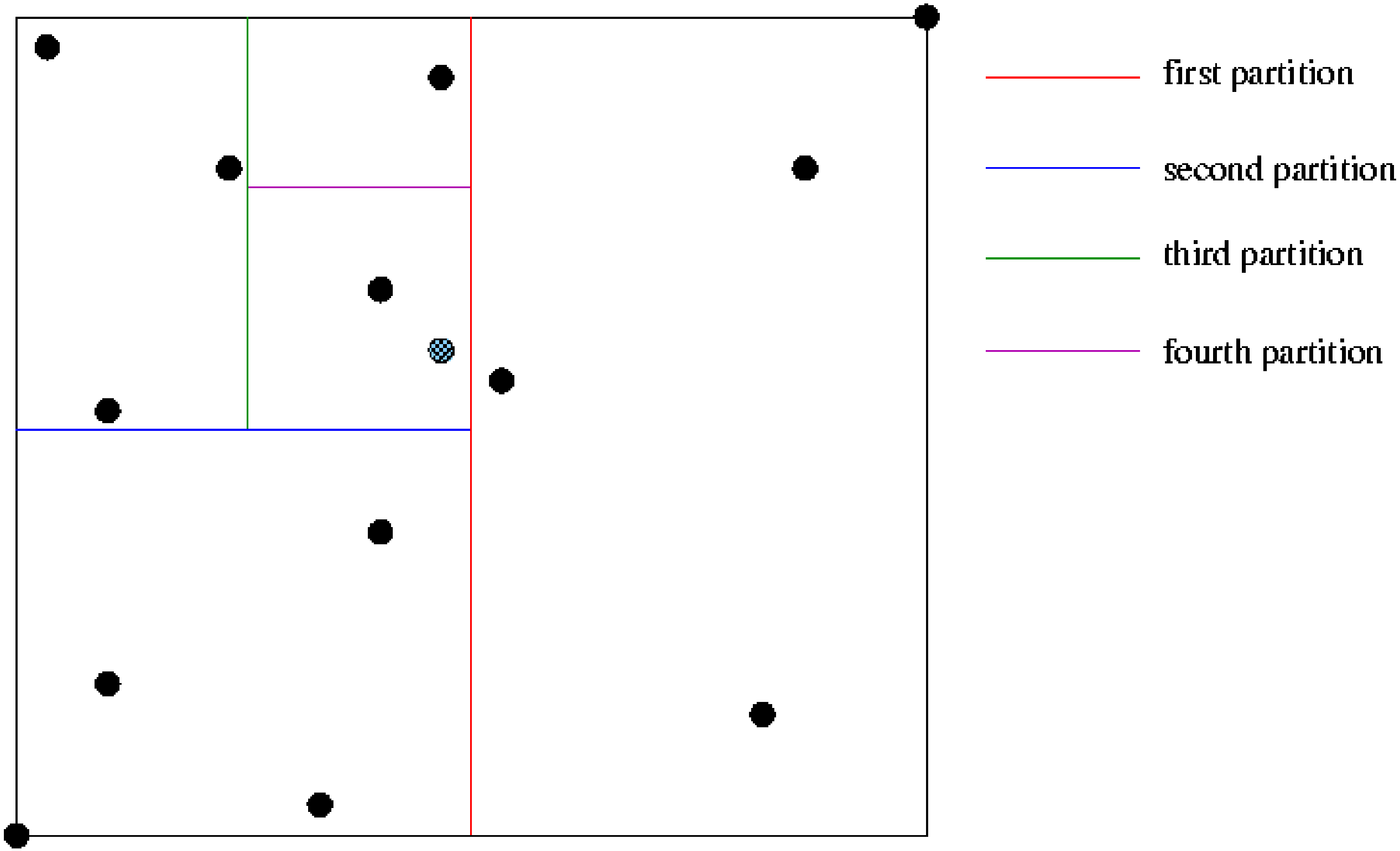
6D SLAM – The ICP Algorithm (10)



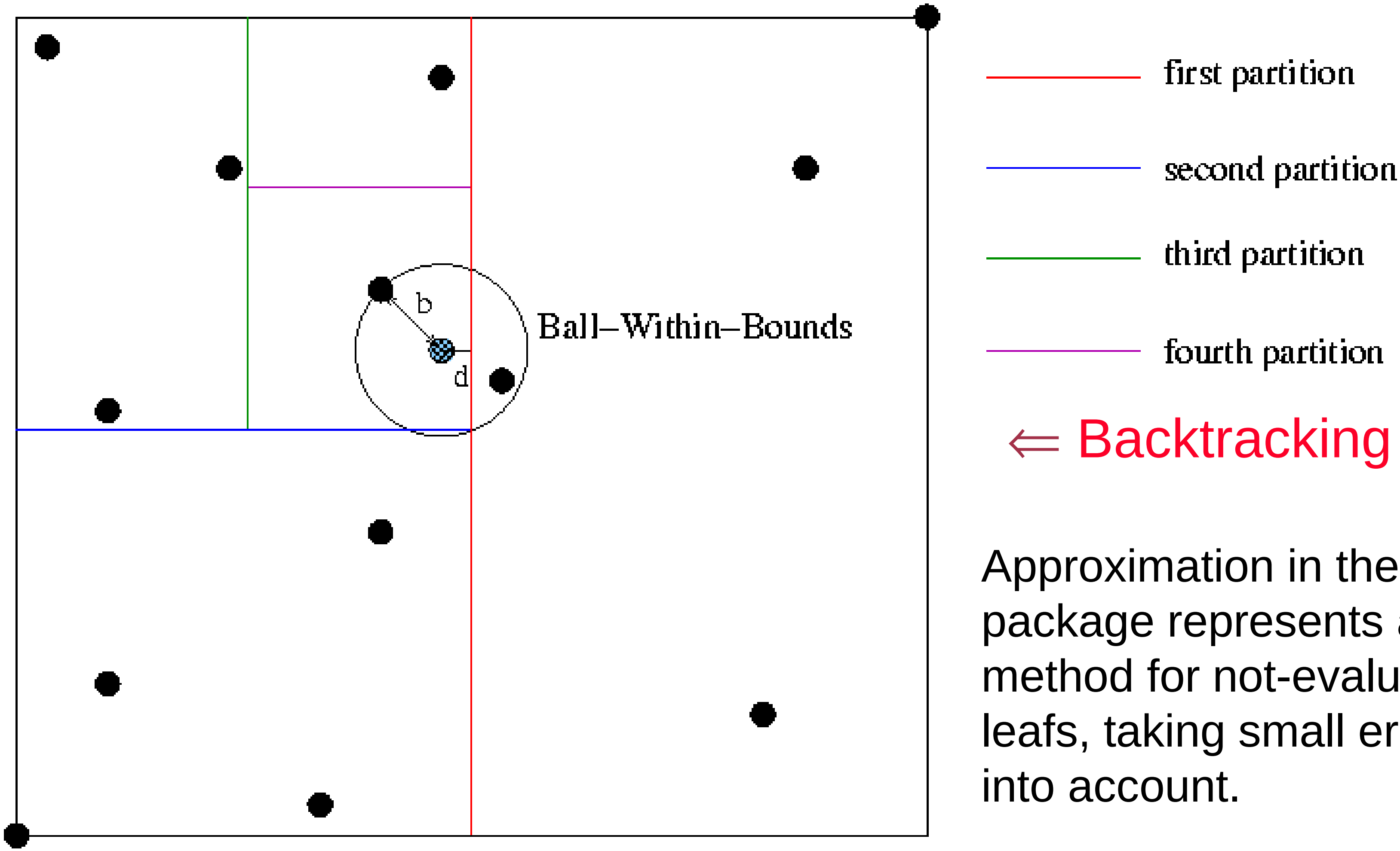
6D SLAM – The ICP Algorithm (11)



6D SLAM – The ICP Algorithm (12)



6D SLAM – The ICP Algorithm (13)

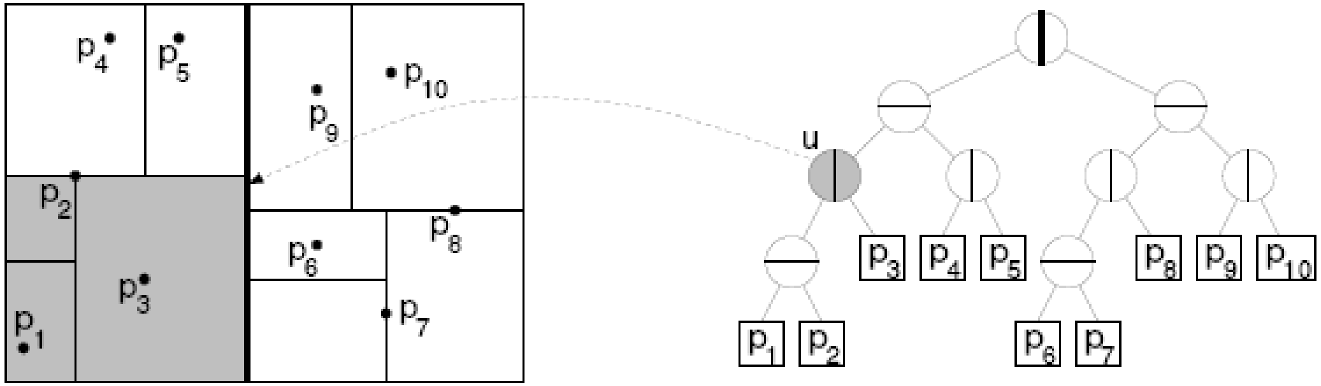


⇐ Backtracking

Approximation in the ANN package represents a method for not-evaluating leafs, taking small errors into account.

6D SLAM – The ICP Algorithm (14)

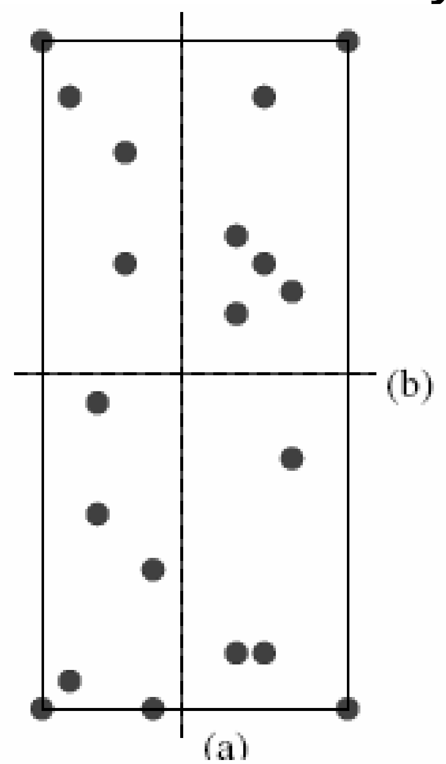
- How to split a k-d tree during construction?



1. Splitting at median
   - Fast calculation of median is needed (accomplishable in O(n))
   - Cells may have an arbitrary aspect ratio
   - Final tree has size $\lceil \log_2 n \rceil$
2. Midpoint splitting rule
   - Fast and easy to compute
   - Guarentees aspect ratio, but may result in trivial splits
3. Midpoint splitting rule that reverts to splitting at media to avoid degeneration.

6D SLAM – The ICP Algorithm (15)

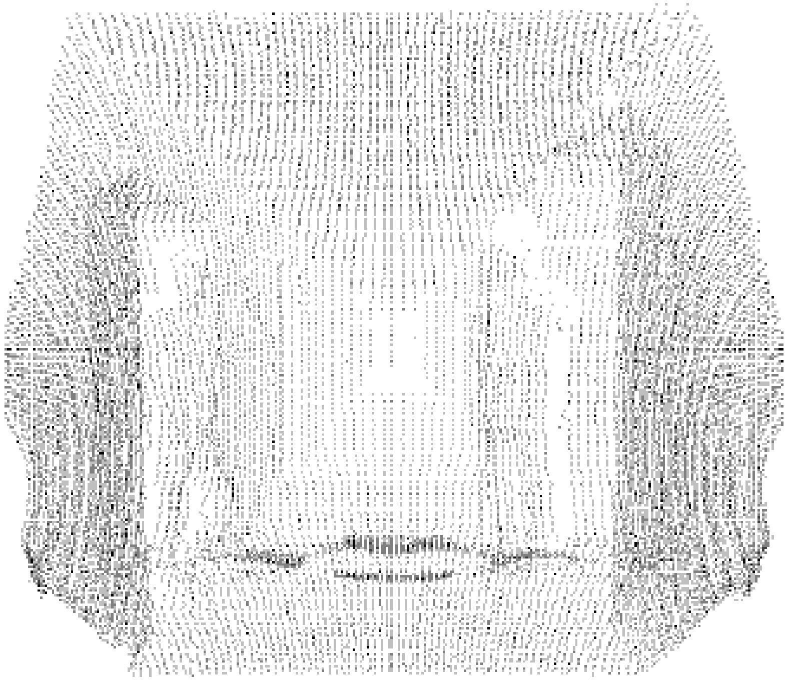
- Other methods are implemented in ANN as well
- Best performance is achieved by the so-called optimized k-d tree

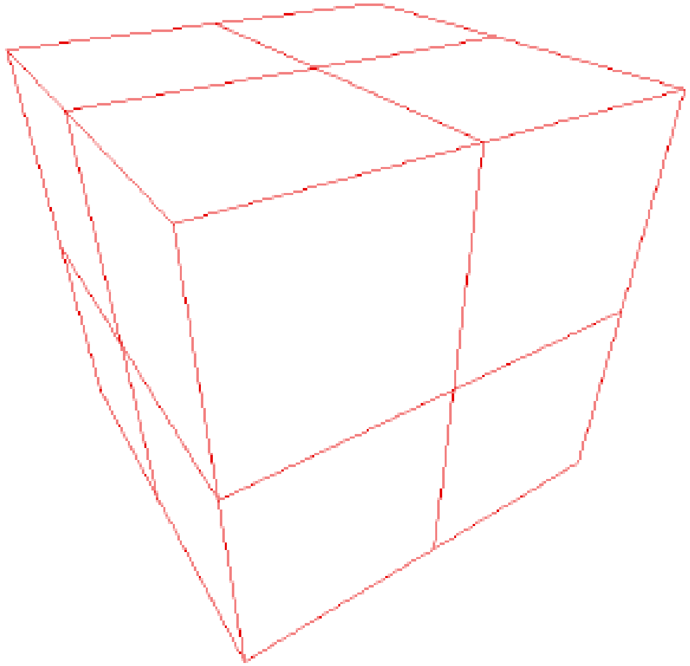


Choose (b) over a, since it reduces the total amount of backtrackung.

6D SLAM – The ICP Algorithm (16)

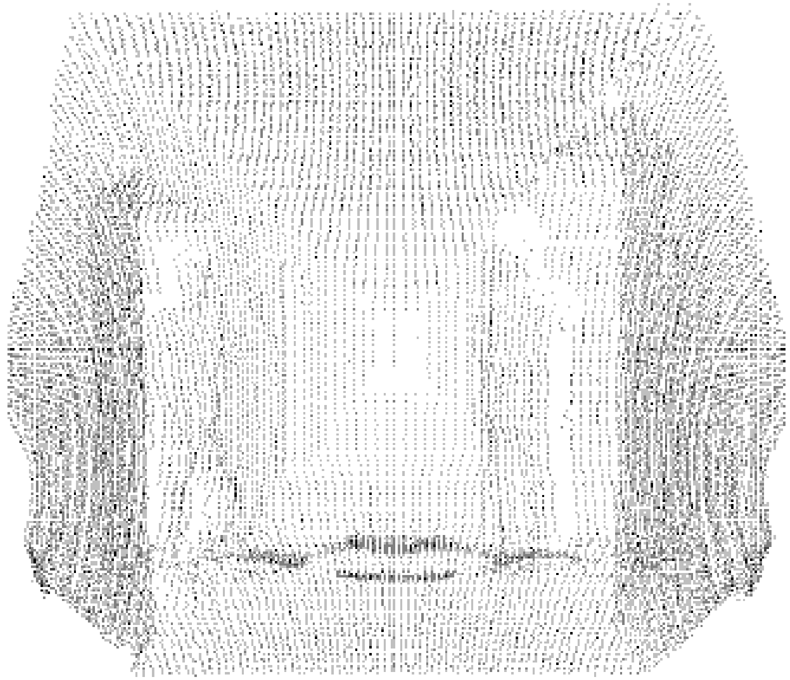
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

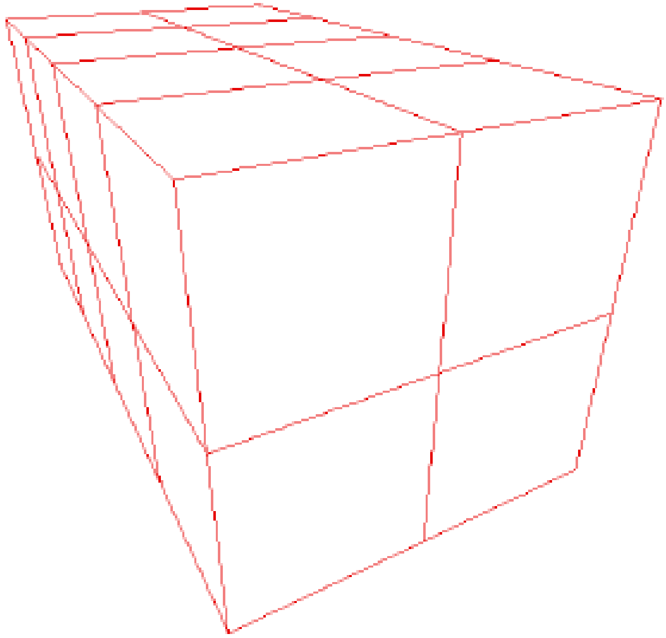




6D SLAM – The ICP Algorithm (17)

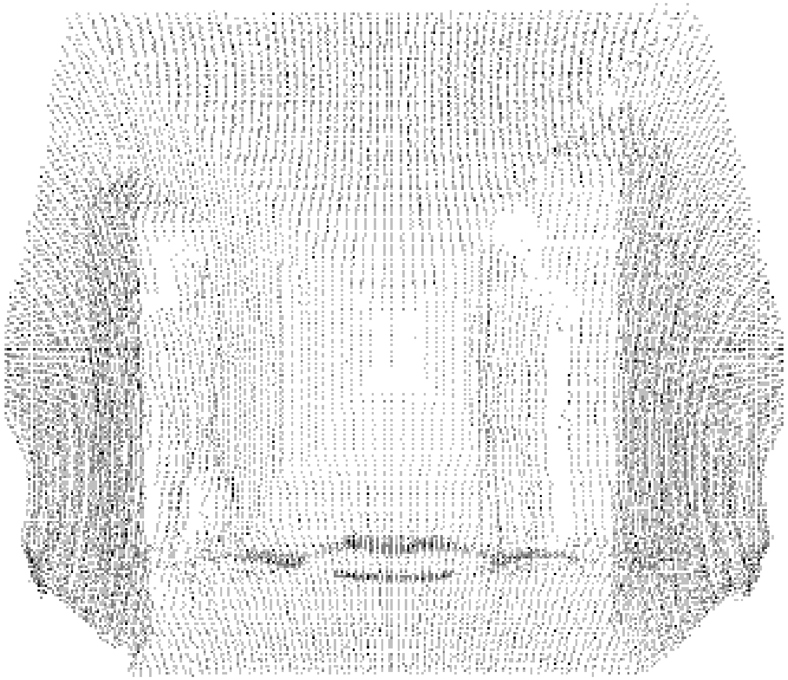
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

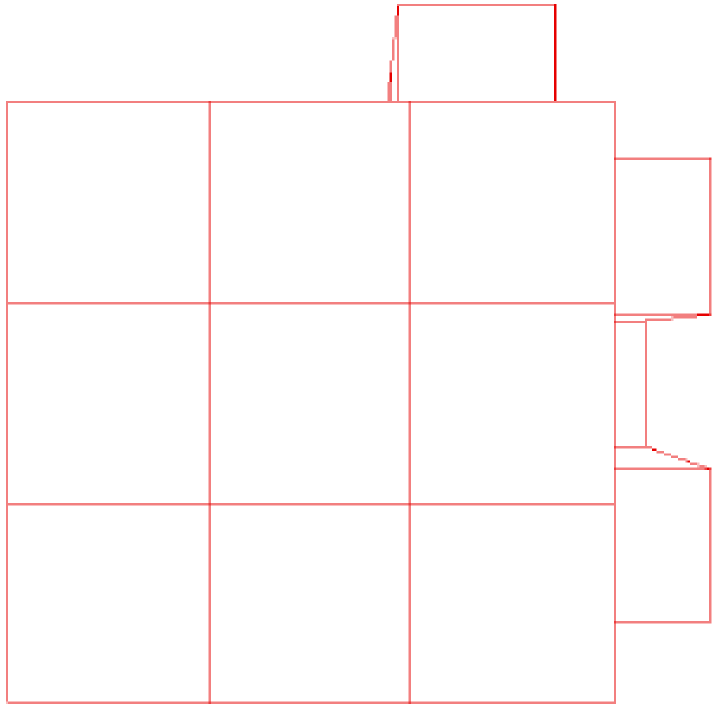




6D SLAM – The ICP Algorithm (18)

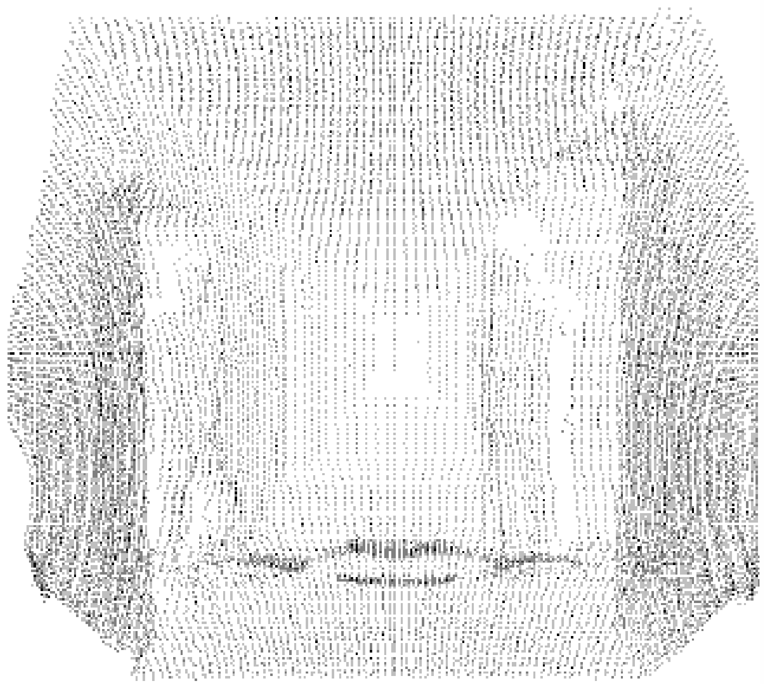
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

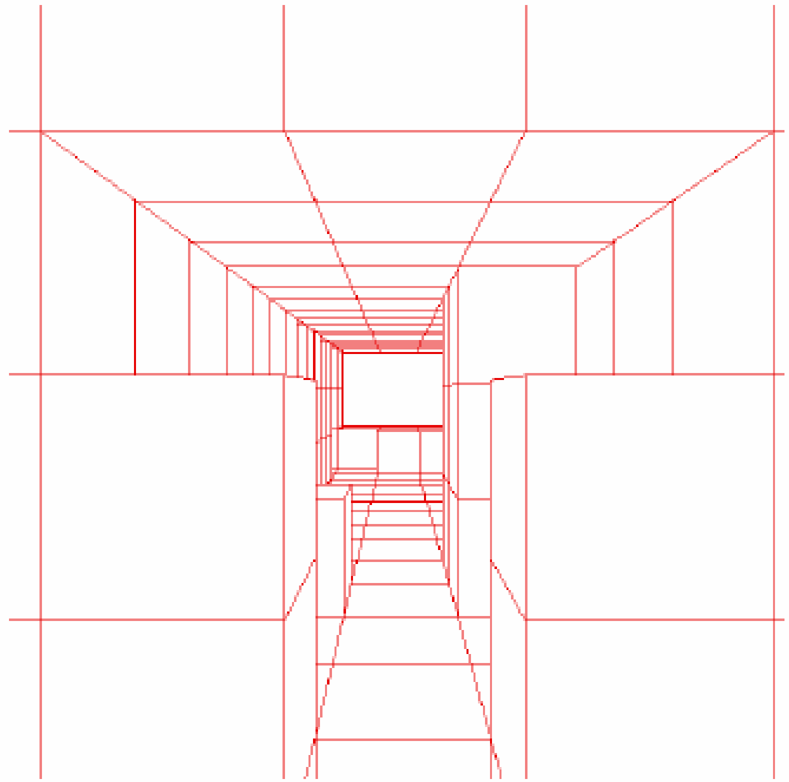




6D SLAM – The ICP Algorithm (19)

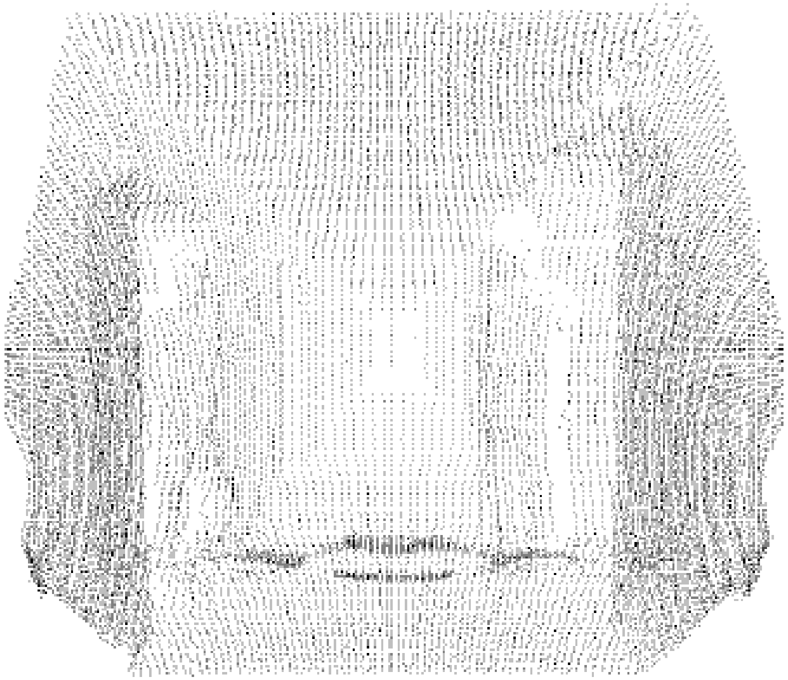
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

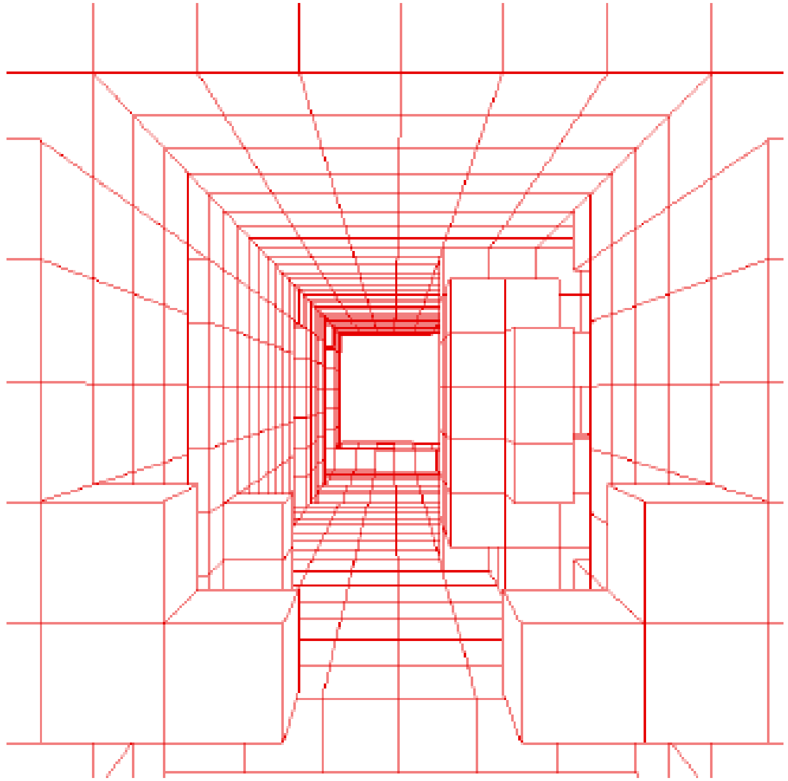




6D SLAM – The ICP Algorithm (20)

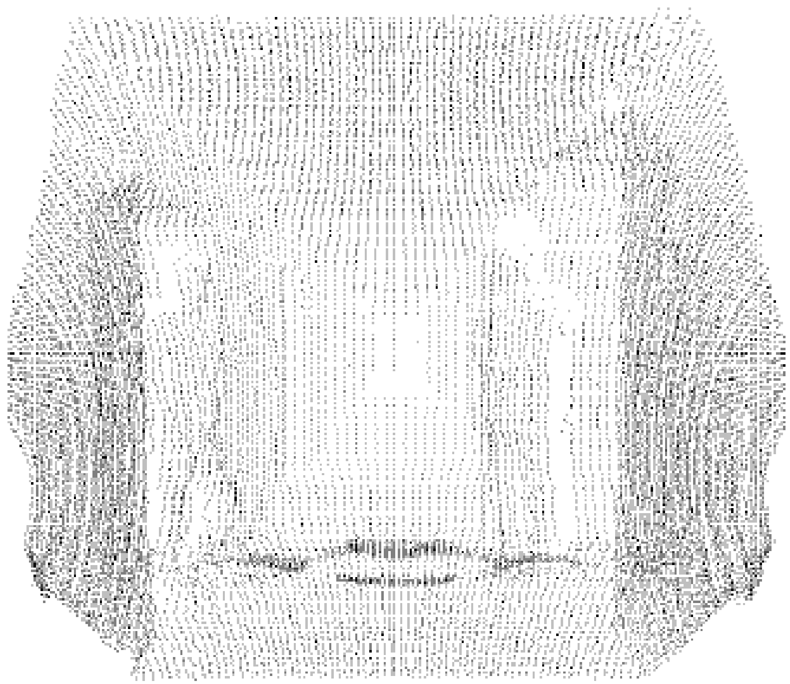
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

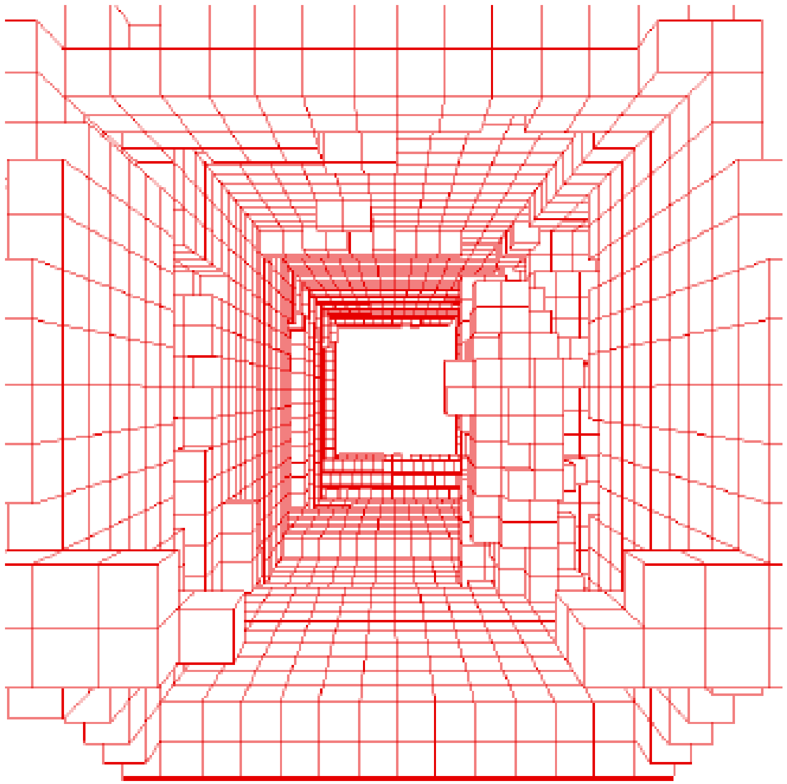




6D SLAM – The ICP Algorithm (21)

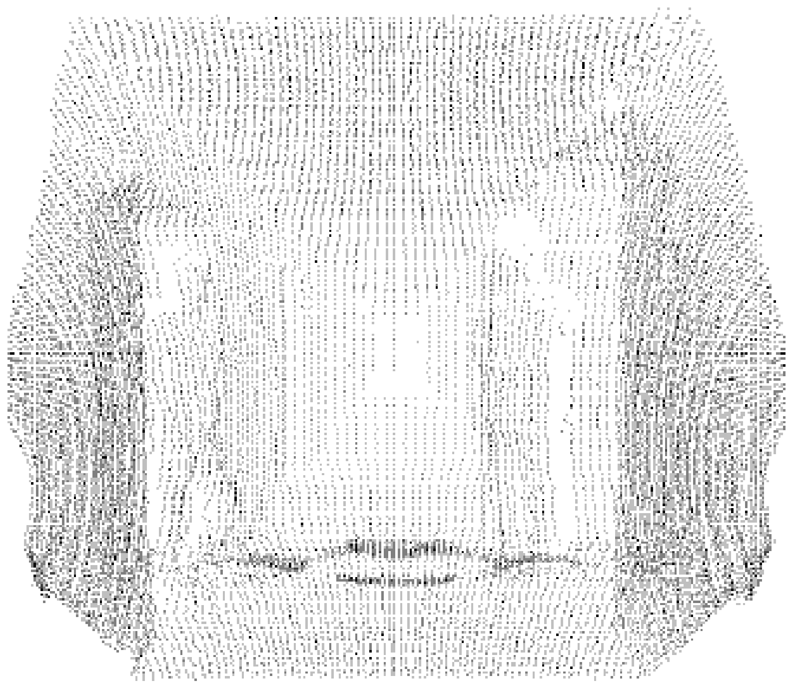
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

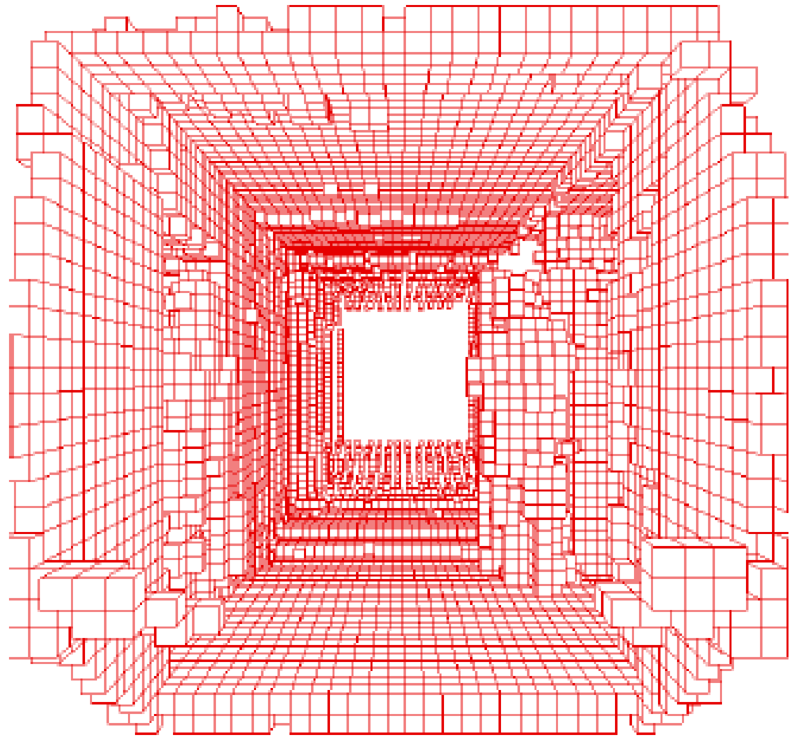




6D SLAM – The ICP Algorithm (22)

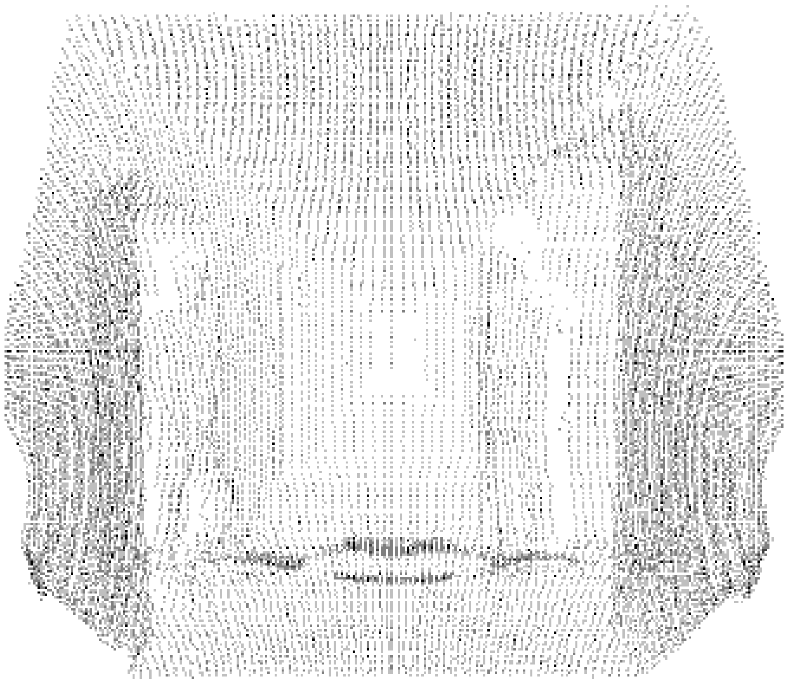
- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

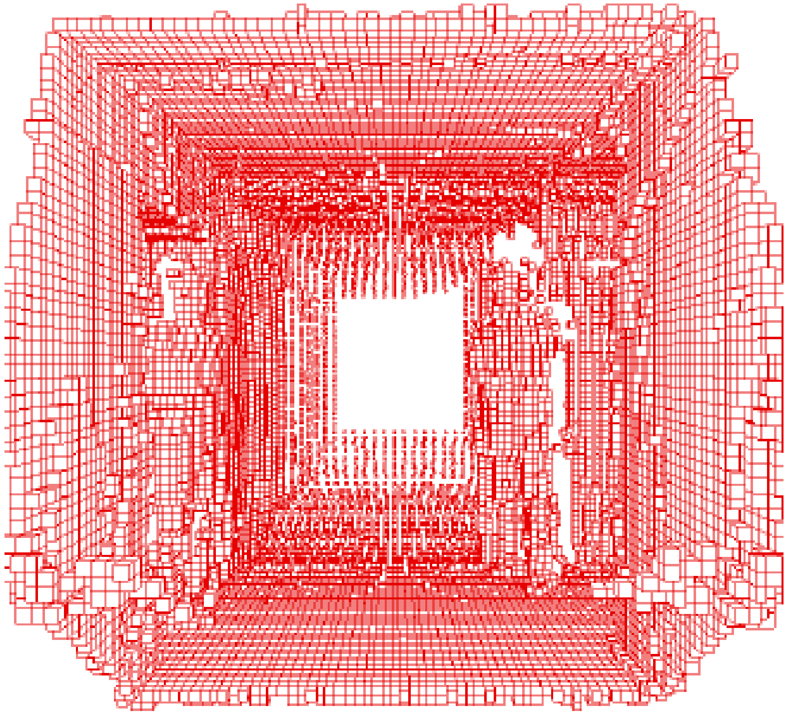




6D SLAM – The ICP Algorithm (23)

- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud

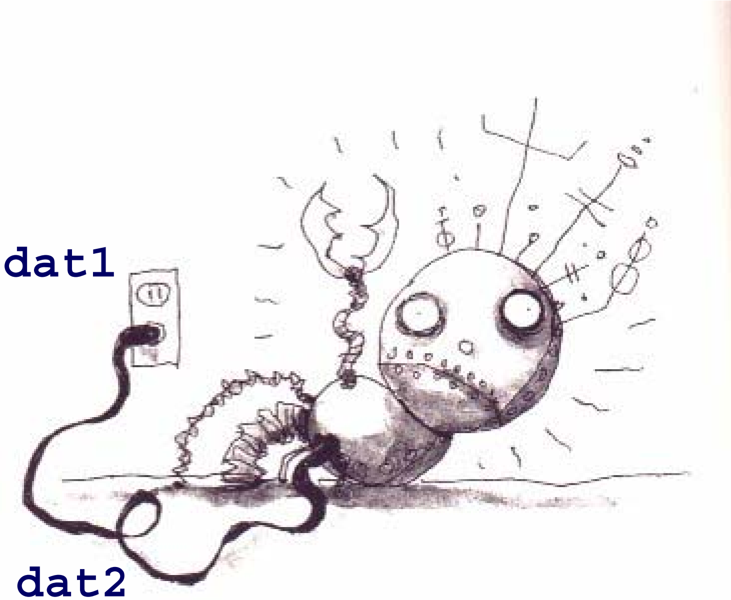




6D SLAM – Hands-on-experience (2)

- Things to try
  - Odometry extrapolation and ICP on outdoor 3D data

```
bin/slam6D -s 0 -e 3 -m 3000 --algo=2 dat1
bin/show -s 0 -e 3 -m 3000 dat1
```



  - Odometry extrapolation and ICP on indoor data

```
bin/slam6D -s 0 -e 15 -m 3000 --algo=2 dat2
bin/show -s 0 -e 15 -m 3000 dat2
```

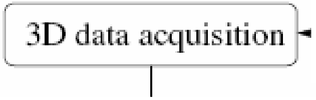
  - Odometry extrapolation and ICP using reduced data (indoors)

```
bin/slam6D -s 0 -e 15 -m 3000 --algo=2 -r 10 dat2
bin/show -s 0 -e 15 -m 3000 dat2
```

  - Odometry extrapolation and ICP on a large loop (Univ. Hannover)

```
bin/slam6D -s 0 -e 75 -r 10 -i 100 --epsICP=0.00001 -d 150 --lastscan dat3
bin/show -s 0 -e 75 dat3
```

6D SLAM – Global Relaxation (1)



6D SLAM – Global Relaxation (1)

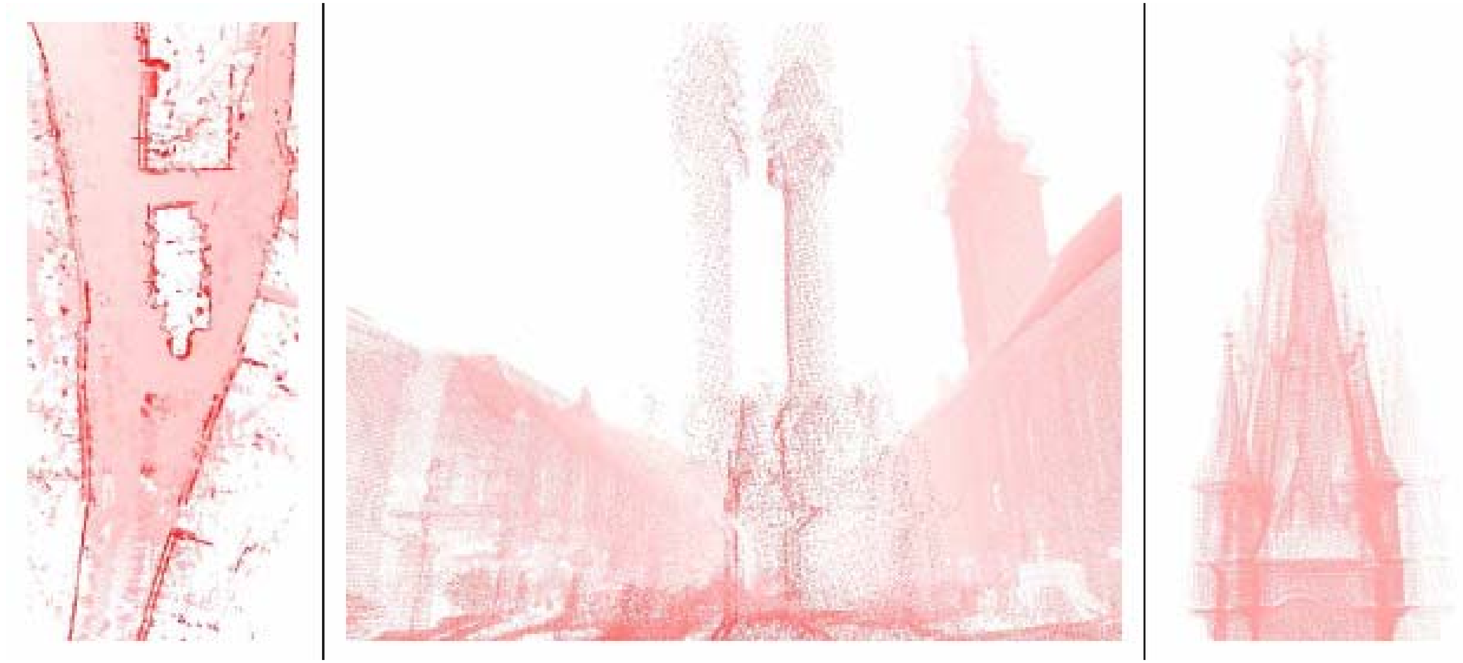
- In SLAM loop closing is the key to build consistant maps
  - Notice: Consistant vs. correct or accurate

- GraphSLAM
  1. Graph Estimation
  2. Graph Optimization

1. Graph Estimation
   - Simple strategy, connect poses with graph edges that are close enough

6D SLAM – Global Relaxation (1)

- Consecutive Scan Matching with ICP results in an erronous map, since small matching errors sum up.

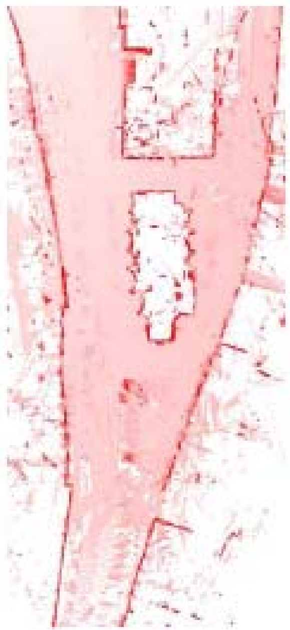


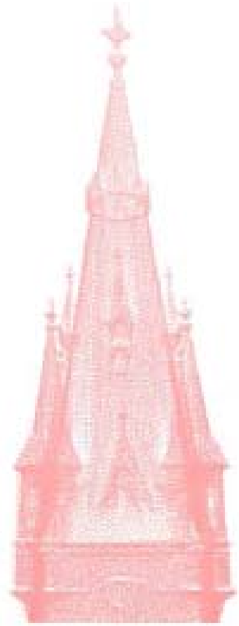
6D SLAM – Global Relaxation (2)

- Consecutive Scan Matching with ICP results in an erronous map, since small matching errors sum up.

6D SLAM – Global Relaxation (3)

- Consecutive Scan Matching with ICP results in an erronous map, since small matching errors sum up.





6D SLAM – Global Relaxation (4)

- Consecutive Scan Matching with ICP results in an erronous map, since small matching errors sum up.

⇨ Replace the ICP error function by a global one, i.e.,

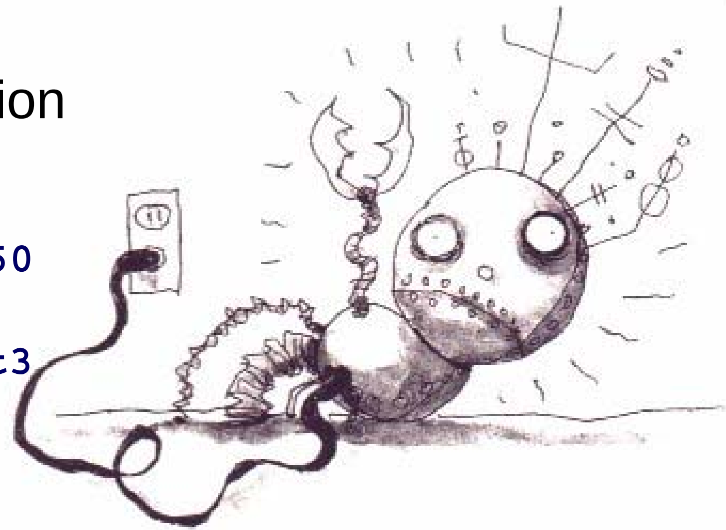
$$D_{i,j} = X_i - X_j$$

$$W = \sum_{(i,j)} (D_{i,j} - \bar{D}_{i,j})^T C_{i,j}^{-1} (D_{i,j} - \bar{D}_{i,j})$$

where $\bar{D}_{i,j} = D_{i,j} + \Delta D_{i,j}$ models random Gaussian noise, added to the unkown exact pose $D_{i,j}$ and $C_{i,j}$ the co-variance matrix of the overlapping scans computed from **closest point pairs**.

6D SLAM – Hands-on-experience (3)

- Things to try
  - Odometry extrapolation and ICP and and loop detection and global relaxation on a large loop



```
bin/slam6D -s 0 -e 75 -r 10 -i 100 -d 250
           --epsICP=0.00001 --lastscan
           -I 50 --cldist=750 -D 500 dat3
bin/show -s 0 -e 74 dat3
```

  - Closed loop detection, even larger data set (Univ. of Hannover)

```
bin/slam6Dr -s 22 -e 324 -r 10 -i 100 -d 250
            --epsICP=0.00001 --lastscan
            -I 25 --cldist=500 dat_hannover
bin/showr -s 22 -e 323 dat_hannover
```